Secure and Efficient Two-Party Signing Protocol for the Identity-Based Signature Scheme in the IEEE P1363 Standard for Public Key Cryptography

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Abstract—Mobile device and application (app) security are increasing important, partly due to the constant and fast-paced cyberthreat evolution. To ensure the security of communication (e.g., data-in-transit), a number of identity-based signature protocols have been designed to facilitate authorization identification and validation of messages. However, in many of these protocols, a user’s private key may leak when a new signature is generated since the private keys are stored on the device. Seeking to improve the security of the private key, we propose a novel two-party distributed signing protocol for the IEEE P1363 standard in this paper. This protocol requires two devices to separately store one part of the user’s private key, and allows these two devices to generate a valid signature without revealing the entire private key of the user. We formally prove the security of the protocol in the random oracle model. Then, we implement the protocol using the MIRACL library and evaluate the protocol on two Android devices. Compared with the protocol of Lindell (CRYPTO’17) that uses zero-knowledge for its security, we demonstrate that our protocol is more suitable for deployment in the mobile environment.

Index Terms—Two-party signature, mobile device, secure and efficient

1 INTRODUCTION

MOBILE device and application (app) popularity is perhaps best illustrated by their market penetration rate. According to a research released by the China Internet Network Information Center in December 2016 [1], for example, the number of mobile users in China is estimated to be 695 million, among them the number of Internet users has increased by 95.1 percent compared to the corresponding period of last year. A year later in December 2017, the number of mobile users in China has increased by 2.4 percent compared to the year before [2]. This is not surprising, partly due to many technological trends (e.g., Internet of Things - IoT, mobile cloud, and mobile social networking) and advances in mobile technologies [3], [4], [5], [6], [7]. For example, a recent Gartner research report [8] posited that:

In the next stage of mobility, the mobile user will be increasingly enveloped in a growing “device mesh” the expanding set of endpoints people use to access applications and information or to interact with people, social communities, governments and businesses. This will also involve an increasing number of devices (such as wearables, drones and the Internet of Things (IoT)), enabling technologies (such as 5G, wireless power and Bluetooth) and interface modalities (such as augmented and virtual reality and natural-language processing [NLP]).

A typical mobile Internet architecture is shown in Fig. 1, where mobile (Internet-connected) devices can collect, transmit, process and store a wide range of data (e.g., environmental data, and personally identifiable information such as health data). To ensure the security of such significant amount of data, we have to also improve the security of mobile devices and apps.

One of the challenges in designing security solutions, particularly cryptographic solutions, is the diversity in mobile device hardware and software (e.g., operating system and apps) and their computing capabilities (e.g., some mobile devices have limited computation and storage capabilities) [9]. In a typical network, communications between mobile and other devices are subject to a broad range of attacks,
varying from basic attacks to sophisticated advanced persistent threat-type attacks [10], [11], [12]. Therefore, it is important to verify the identity of the mobile device and confirm the validation of the message. To achieve this goal, we generally rely on the technology of digital signature schemes [13], [14]. However, in the most schemes, the user’s private key is typically stored as a single file on the mobile device (i.e., single point of attack) [15]. There is also the risk that the private key can be obtained during the signature generation process, or through the use of mobile forensic techniques [16].

Threshold cryptography [17], [18], [19], [20] can potentially be used to improve the security of the private key, by splitting and distributing a single key into a number of shares/parts that can then be stored on different devices (or for multiple users). Threshold cryptography has been intensively studied since the late 1980s [21], [22], [23], [24]. One common approach is the \( t \)-out-of-\( n \) threshold secret sharing protocol proposed by Shamir [25] and Blakley [26], where a single (private) key is shared among \( n \) parties. This allows the recovery of the private key with \( t \) or more shares, rather than all shares in the original threshold cryptography-based approach. In other words, to obtain the private key, an adversary has to corrupt at least \( t \) parties or devices. In the context of this paper, an adversary must corrupt all the devices in order to reconstruct and obtain the private key. Such an approach can be used in applications where generating a signature requires multiple users to sign the message or a ciphertext should be decrypted by multiple users.

A known limitation in the \( t \)-out-of-\( n \) threshold secret sharing protocol is that the (entire) private key can be recovered during the signing or encryption phase. As the recovered key is usually stored on the mobile devices (e.g., Android or iOS devices), any party who holds the recovered private key can sign or encrypt message without requiring the participation of other parties. Recently, Lindell [27] designed a fast secure two-party ECDSA signing protocol in CRYPTO 2017. However, the use of zero-knowledge in the security proof may mean that it is not practical/valid for deployment in a mobile environment.

In this paper, we propose a two-party distributed signature protocol for the IEEE P1363 standard. Specifically, in our proposed protocol, the two parties communicate with each other and one of them outputs a signature without revealing the private key. We also implement the proposed protocol on two mobile devices and a personal computer for evaluation. The main contributions of our work are summarized as follows:

1) We propose a fast and secure two-party distributed signing protocol for the identity-based signature scheme in IEEE P1363. The two parties in the protocol can generate a valid signature without recovering the private key.
2) We analyze the security of our proposed protocol. The analysis shows that our protocol can satisfy the security requirements when implemented on two devices. Our scheme is secure if computing the DL problem, CDH problem and k-CAA problem are infeasible.

We analyze the security of our proposed protocol. The experimental results show that our protocol is efficient and practical in real-world applications.

In Sections 2 and 3, we review related literature on threshold secret sharing protocol and background materials (i.e., bilinear pairings, IEEE standard for identity-based signature protocol, and the underpinning mathematical assumptions), respectively. In Section 4, we present our two-party distributed signing protocol. We then analyzing its security in Section 5, and its performance in Section 6. Finally, we conclude this paper in the last section.

2 RELATED WORK

Shamir [25] and Blakley [26] independently introduced the concept of threshold secret sharing to protect the security of (private) keys, and designed the first two such protocols. However, cheaters can attempt to beat the system by presenting fake shares, resulting in honest shareholders obtaining a fake secret. Shamir’s protocol is not capable of detecting such a malicious user in the process of secret reconstruction. Thus, Harn and Lin [28] extended Shamir’s protocol to detect and identify cheater utilizing redundant shares. In their approach, it is assumed that there are more than \( t \) participants in the secret reconstruction.

In a follow-up work, Tian et al. [29] presented a fair \( (t, n) \) threshold secret sharing protocol using the cheating detection proposed by Harn and Lin [28]. In their approach, the authors studied the fairness of secret reconstruction and demonstrated that their protocol achieves security and fairness against three attacks. Two of the three attacks are relevant to synchronous network, and one attack is relevant to asynchronous network. However, Harn [30] pointed out that the protocol of Tian et al. [29] can only be deployed in a synchronous network, contrary to the authors’ claim. Harn also pointed out that Tian et al.’s protocol is vulnerable to impersonation attacks performed by an external adversary, since this adversary can send a forged valid share to other users after \( t \) shares are released.

Tompa and Woll [31] designed a protocol in which the secret message is hidden in the same false secret sequence. The cheats have an advantage over a honest user in guessing the position of the real secret in sequence. Lee and Laith [32] proposed a V-fairness \( (t, n) \) secret sharing protocol, where they defined the number of \( v \) (i.e., the number cheats) to be less than half of \( t \). Thus, all players have an equal chance of revealing the secret key when they release their shares simultaneously. Hwang and Chang [33] proposed an improved protocol, in which they used twice cheater detection method to enforce each player having only two shares. However, the private key can be recovered during the signing or encryption phase.

3 PRELIMINARIES

A summary of notations used in this paper is presented as follows.

\( U_i \): User
\( P_1, P_2 \): two devices for \( U_i \)
Input: a user’s identity ID
Output: a public parameter set P

KGC

\[ t_1 = s + H_1(ID) \pmod{q} \]
\[ t_2 = t_2^{-1} \pmod{q} \]
Generate \( d_1, d_2 \in [1, q - 1] \)
\[ d_1 \cdot d_2 = t_2 \pmod{q} \]
\[ D_0^b = d_1 Q_1 \]
Partial private key \( g_t = g^{d_1^2} \)
\[ D_0^b = (d_2, g_t) \]

**Fig. 2. Key generation.**

- \( q \): the order of group
- \( s \): the master key of KGC
- \( G_1, G_2 \): additive cyclic groups of order \( q \)
- \( G_T \): multiplicative cyclic group of order \( q \)
- \( Q_1, Q_2 \): the generators of additive cyclic groups \( G_1, G_2 \)
- \( D_{ID} \): private key of user \( U_i \)
- \( D_{ID}^1, D_{ID}^2 \): partial private key of two devices \( P_1, P_2 \)
- \( H_1, H_2 \): hash function of \( \{0, 1\}^* \rightarrow Z_q^* \)
- \( e \): bilinear pairing of \( G_1 \times G_2 \rightarrow G_T \)
- \( g \): \( e(Q_1, Q_2) \)
- \( g^t \): exponentiation
- \( [x, y] \): number set between \( x \) and \( y \)

### 3.1 Bilinear Pairing

Let \( G_1 \) and \( G_2 \) denote additive cyclic groups, \( G_T \) denote a multiplicative cyclic group, and \( e : G_1 \times G_2 \rightarrow G_T \) denotes a bilinear map. Suppose \( Q_1 \) and \( Q_2 \) are the generators of \( G_1 \) and \( G_2 \), \( g \) is the element that \( Q_1 \) and \( Q_2 \) maps to \( G_T \). Thus, the map \( e \) is a bilinear pairing on the condition that \( e \) satisfies the following properties:

- **Bilinear:** Given any two elements \( a, b \in Z_q^* \), and \( \forall X \in G_1, \forall Y \in G_2 \), there is \( e(a \cdot X, b \cdot Y) = e(X, Y)^{ab} \)
- **Non-degenerate:** There exists at least one element \( X \) satisfies the inequation \( e(X, X) \neq 1 \)
- **Efficient Computability:** Given any two elements \( \forall X \in G_1, \forall Y \in G_2 \), there exists at least one efficient algorithm to compute \( e(X, Y) \).

### 3.2 IEEE Standard for Identity-Based Signature

We briefly review the IEEE standard for identity-based signature scheme (BLMQ signature scheme) [34]. The steps of signature generation are as follows:

#### 1. Setup

Input: a security parameter \( k \)
Output: a public parameter set \( P \)

(a) Establishes the cyclic groups \( G_1, G_2, G_T \) and a bilinear map \( e : G_1 \times G_2 \rightarrow G_T \)

(b) Selects two random generators \( Q_1 \in G_1 \) and \( Q_2 \in G_2 \)

(c) Generates a random server secret \( s \) in \( Z_q^* \), then calculates \( P_{pub} = s \cdot Q_2 \) and \( g = e(Q_1, Q_2) \).

(d) Makes the set \( P = \{P_{pub}, g, Q_1, Q_2, G_1, G_2, G_3, e\} \).

#### 2. Extract

Input: a user’s identity ID, a public parameter set \( P \) and the server secret \( s \)
Output: user’s private key \( D_{ID} \)

(a) Computes the identity element \( h_{ID} = H_1(ID) \) in \( Z_q^* \)

(b) Computes \( D_{ID} = (s + h_{ID})^{-1}Q_1 \).

### 3. Sign

Input: a message \( m \), a public parameter set \( P \) and user’s private key \( D_{ID} \)
Output: a signature \( \sigma \)

(a) Generates a random number \( r \in Z_q^* \) and computes \( u = g^r \)

(b) Computes \( h = H_2(m, u) \) and \( S = (r + h) \cdot D_{ID} \)

(c) Outputs signature \( \sigma = (h, S) \).

### 4. Verify

Input: a signature \( \sigma \), a message \( m \) and user’s public key \( h_{ID} \)
Output: valid if the signature is accepted, otherwise output invalid

(a) Computes \( u = g^h \)

(b) If \( h = H_2(m, u) \), then outputs valid, otherwise outputs invalid.

### 4 Proposed Two-Party Signing Protocol

In this section, we describe the two-party signing protocol, which consists of the key generation phase and the distributed signature generation phase.

#### 4.1 Key Generation Phase

In the key generation phase (see Fig. 2), KGC generates the user’s private keys \( D_{ID}^1 \) and \( D_{ID}^2 \). Then, KGC distributes \( D_{ID}^1 \) and \( D_{ID}^2 \) to be stored on two devices \( P_1 \) and \( P_2 \), respectively.

1. \( U_i \) sends a request (i.e., user ID ID) to KGC.
2. Upon receiving the request, KGC computes \( t_1 = s + H_1(ID) \) and \( t_2 = t_2^{-1} \pmod{q} \).
3. KGC generates two random numbers \( d_1 \in [1, q - 1] \) and computes \( d_2 = d_1 \cdot d_2^{-1} \pmod{q} \).
4. KGC computes the first partial private key \( D_{ID}^1 = d_1 \cdot Q_1 \) and sends \( D_{ID}^1 \) to \( P_1 \).
5. Then, KGC computes a partial private key \( g_t = g^{d_1^2} \)
   and the second partial private key \( D_{ID}^2 = (d_1, g_t) \).
6. Finally, KGC sends \( D_{ID}^2 \) to \( P_2 \).
7. Notice that, the user’s private key \( D_{ID} = d_2 D_{ID}^1 \).

#### 4.2 Distributed Signature Generation Phase

In the distributed signature generation phase (see Fig. 3), the user inputs the message \( m \) to be signed and receives a legitimate signature \( \sigma = (h, S) \) as the output.

1. \( P_1 \rightarrow P_2: \{\text{request}\} \)
   \( P_1 \) sends a signature request to \( P_2 \).
2. \( P_2 \rightarrow P_1: \{\mu_1, \mu_2\} \)
   (a) Upon receiving the request from \( P_1 \), \( P_2 \) chooses two random numbers \( k_1, k_2 \in [1, q - 1] \).
   (b) \( P_2 \) computes \( \mu_1 = g^{k_1} \) and \( \mu_2 = g^{k_2} \).
   (c) \( P_2 \) sends \( \{\mu_1, \mu_2\} \) to \( P_1 \).
3. \( P_1 \rightarrow P_2: \{h'\} \)
   (a) Upon receiving the message, \( P_1 \) chooses two random numbers \( k_3, q_4 \in [1, q - 1] \).
   (b) \( P_1 \) computes \( h' = h + k_4 \pmod{q} \).

Notice that, the user’s private key \( D_{ID} = d_2 D_{ID}^1 \).
We define the mathematical assumptions required in our security proof.

\[ P_1 \quad D_{ID}^1 = d_1 Q_1 \]

\[ D_{ID}^2 = (d_2, g_1) \quad P_2 \]

Choose \( k_1, k_2 \in [1, q - 1] \)
\[
\mu_1 = g^{k_1}
\mu_2 = g^{k_2}
\]

Choose \( k_3, k_4 \in [1, q - 1] \)
\[
\mu = \mu_1^{k_3} \cdot \mu_2 \cdot d_2
\]
\[
h = H_2(m, \mu)
\]
\[
h' = h + k_4 \pmod q
\]
\[
s_1 = k_1 \cdot d_2 \pmod q
\]
\[
s_2 = (h' + k_2) \cdot d_2 \pmod q
\]
\[
S = s_1 k_3 Q_1 + s_2 D_{ID}^1\]

Output \((h, S)\)

\[ \text{Fig. 3. Distributed signature generation.} \]

(c) \( P_1 \) sends \( \{h'\} \) to \( P_2 \).

4. \( P_2 \rightarrow P_1: \{s_1, s_2\} \)

(a) Upon receiving the message, \( P_2 \) computes \( s_1 = k_1 \cdot d_2 \pmod q \) and \( s_2 = (h' + k_2) \cdot d_2 \pmod q \).

(b) \( P_2 \) sends \( \{s_1, s_2\} \) to \( P_1 \).

5. \( P_1 \) outputs the signature

(a) \( P_1 \) receives the message, and computes \( S = s_1 k_3 Q_1 + s_2 D_{ID}^1\).

(b) \( P_1 \) outputs the signature \((h, S)\).

4.3 Correctness

It is easy for \( P_1 \) to compute \( \mu = g^{k_3 k_1 d_1^{-1} + k_2 + k_4} \) and

\[
S = k_3 \cdot s_1 Q_1 + s_2 D_{ID}^1
\]

\[
= k_1 \cdot k_3 \cdot d_2 Q_1 + (h + k_4 + k_2) \cdot d_2 D_{ID}^1
\]

\[
= k_1 \cdot k_3 \cdot d_2 \cdot d_1^{-1} D_{ID}^1 + (h + k_4 + k_2) D_{ID}^1
\]

\[
= (k_1 \cdot k_3 \cdot d_1^{-1} + k_2 + k_4) D_{ID}^1.
\]

Therefore, the correctness of the proposed protocol for the identity-based signature scheme in the IEEE P1363 standard is demonstrated.

4.4 Verify

When a verifier receives the signature \((h, S)\) on a message \(m\), the following actions are undertaken.

1. Computes \( \mu^* = \frac{e(S, H_1(ID) Q_2 + P_{pub})}{g^h} \).

2. Computes \( h^* = H_2(m, \mu^*) \).

3. If and only if \( h^* = h \), outputs \textit{valid}, otherwise outputs \textit{invalid}.

5 SECURITY ANALYSIS

5.1 Mathematical Assumptions

We define the mathematical assumptions required in our security proof.

\textbullet \textbf{Discrete Logarithm (DL) Problem:} Let \( G \) be cyclic groups of prime order \( q \). The DL problem in \( G \) is to compute \( a \in Z_q \) for any given \( P, Y = aP \in G \). An probability polynomial time (P.P.T) algorithm \( A \) has advantage \( \epsilon \) in solving DL in \( G \) if

\[
Pr[A(P, Y) = a: a \in Z_q, Y = aP] \geq \epsilon.
\]

We say that DL problem in \( G \) is infeasible if all polynomial time (P.P.T) algorithms have a negligible advantage \( \epsilon \) in solving DL in \( G \).

\textbullet \textbf{Computational Diffie-Hellman (CDH) Problem:} Let \( G \) be cyclic groups of prime order \( q \). The CDH problem in \( G \) is given \( P, aP, bP \in G \) for randomly choose \( a, b \in Z_q \) to compute \( abP \in G \). A P.P.T algorithm \( A \) has advantage \( \epsilon \) in solving CDH in \( G \) if

\[
Pr[A(P, aP, bP) = abP : a, b \in Z_q] \geq \epsilon.
\]

We say that CDH problem in \( G \) is infeasible if all P.P.T algorithms have a negligible advantage \( \epsilon \) in solving CDH in \( G \).

\textbullet \textbf{Collusion Attack with \( k \) traitor (k-CAA) Problem:} Let \( G \) be cyclic groups of prime order \( q \). For an integer \( k \), the number \( x, h_1, h_2, \ldots, h_k \in Z_q \) and a generator \( P \) of \( G \). A k-CAA problem instance is: given \( P, Q = XP \in G \), and \( k \) pairs \( (h_1, (x + h_1)^{-1} P), \ldots, (h_k, (x + h_k)^{-1} P) \), finding a pair \( (h, (x + h)^{-1} P) \) for some \( h \notin h_1, h_2, \ldots, h_k \). A P.P.T algorithm \( A \) has advantage \( \epsilon \) in solving k-CAA problem in \( G \) if

\[
Pr \left[A(P, xP, (h_1, (x + h_1)^{-1} P), \ldots, (h_k, (x + h_k)^{-1} P)) \right] \geq \epsilon.
\]

We say that k-CAA problem in \( G \) is infeasible if any polynomial time algorithm have a negligible advantage \( \epsilon \) in solving k-CAA problem in \( G \).
5.2 System Model

Definition 1. Let $A$ be a P.P.T adversary, and $\pi$ be a digital signature protocol $\pi = \langle \text{Gen}, \text{Sign}, \text{Verify} \rangle$. We define $\text{Sign}_{A, \pi}(1^n)$ as follows.

1. $(pk, sk) \leftarrow \text{Gen}(1^n)$. $A$ runs the $\text{Gen}(1^n)$ algorithm, and outputs a random key pair $(pk, sk)$.
2. $(m^*, \sigma^*) \leftarrow A^{\text{Sign}_{\pi}(1^n)}$. $A$ runs the $\text{Sign}_{\pi}(\cdot)$ algorithm, and outputs $(m^*, \sigma^*)$.
3. The set of all $m$ that $A$ queries to its oracle is $M$. When $m^* \notin M$ and $\text{Verify}_{\pi}(m^*, \sigma^*) = 1$, the experiment outputs $1$.

Definition 2 (Existentially Unforgeable under Choose Message Attacks). For any P.P.T adversary $A$, if there exists a negligible function $\mu$ for every $n$ that satisfies $Pr[\text{Sign}_{A, \pi}(1^n) = 1] \leq \mu(n)$, then the signature protocol $\pi$ is existentially unforgeable under the chosen message attack.

Definition 3. Let $A$ be a P.P.T adversary, and $\pi$ be a digital signature protocol $\pi = \langle \text{Gen}, \text{Sign}, \text{Verify} \rangle$. We define $\text{DistSign}^{\pi}_{A, \Pi}(1^n)$, $b \in \{1, 2\}$ as follows.

1. $(pk, sk) \leftarrow A^{\Pi_{b=1}(1^n)}$.
2. $(m^*, \sigma^*) \leftarrow A^{\Pi_{b=2}(\cdot)}$.
3. The set of all inputs $m \in (\text{sid}, m)$ that $A$ queries to its oracle is $M$, and the identifier $\text{sid}$ should not be previously queried. When $m^* \notin M$, and $\text{Verify}_{\pi}(m^*, \sigma^*) = 1$ and $\text{Verify}_{\pi}$ are as specified in $\pi$, the experiment outputs $1$.

Definition 4 (Secure Two-Party Distributed Signature Scheme). For any P.P.T adversary $A$ and every $b \in \{1, 2\}$, if there exists a negligible function $\mu$ for every $n$ that satisfies $Pr[\text{DistSign}^{\pi}_{A, \Pi}(1^n) = 1] \leq \mu(n)$, then $\Pi$ is a secure two-party protocol for distributes signature generation for $\pi$.

Definition 5 (Blindness). Given two signature records and a signature pair, for any P.P.T adversary $A$, $A$ is not able to specify which signature record belongs to the signature pair, then $\Pi$ is not blind.

In the above definitions, we should first run the distributed key generation phase and only once, before running the distributed signature phase. A honest device $P_{b=1}$ operates the protocol $\Pi$ and makes the stateful oracle $\Pi_{b=1}(\cdot)$. In the two-party signature generation phase $\text{DistSign}^{\pi}_{A, \Pi}$, $A$ can control the device $P_{b}$, $b \in \{1, 2\}$, choose the messages to be signed, and interact with multiple instances to generate a forged signature concurrently.

$A$ can win the game if the message $m^*$ for the forged signature is not queried. The oracle has two inputs: a session identifier and an input, and it works as follows:

1. On receiving a query $(\text{sid}, m)$, if the distributed key generation phase has not completed, then oracle returns $\bot$.
2. In the event that a query $(\text{sid}, m)$ is received when the distributed key generation phase has commenced, if $\text{sid}$ is the first to be queried, then the device $P_{b=1}$ operates the protocol $\Pi$ with session identifier $\text{sid}$ and the input message $m$ which is to be signed. What the device $P_{b=1}$ first sends in the sign phase is the oracle replay.
3. On receiving a query $(\text{sid}, m)$ when the distributed key generation phase has computed and identifier $\text{sid}$ has been queried, the oracle sends the message $m$ to the device $P_{b=1}$ and returns the next message output from $P_{b=1}$.

5.3 Security Proof

We prove the protocol $\Pi$ is a secure two-party distributed signature protocol and existentially unforgeable under chosen message attacks, and the protocol is also blind, in this section.

Theorem 1. Suppose that IEEE standard for identity-based (BLMQ) signature scheme is existentially unforgeable under chosen message attacks, then our protocol is a secure two-party distributed signature and existentially unforgeable under chosen message attacks.

Proof. If an adversary $A$ can break the protocol with the probability $\epsilon$, then it can break the protocol with probability $\epsilon + \mu(n)$, where $\mu$ is a negligible function. In this proof, we separately prove the security of a corrupted $P_{1}$ and a corrupted $P_{2}$. For any $A$ who can attack the protocol and forge a signature in $\text{DistSign}^{\pi}_{A, \Pi}(1^n)$, we select another adversary $S$ who can forge a BLMQ signature in $\text{Sign}_{A, \pi}(1^n)$ with a negligibly close probability. Then, for every P.P.T adversary $A$ and $b \in \{1, 2\}$, there exists a negligible function $\mu$ for every $n$ and an adversary $S$, that satisfies,

$$|Pr[\text{Sign}_{S, \pi}(1^n) = 1] - Pr[\text{DistSign}^{\Pi}_{A}(1^n) = 1]| \leq \mu(n)$$

According to Definition 2, there exists a negligible function $\mu'$ for every $n$ that satisfies $Pr[\text{Sign}_{A, \pi}(1^n) = 1] \leq \mu'(n)$. Combining this with the above equation, we obtain $Pr[\text{DistSign}^{\Pi}_{A, \Pi}(1^n) = 1] \leq \mu(n) + \mu'(n)$. Thus, we prove the equation from two aspects: $b = 1$ and $b = 2$.

Proof of $b = 1$. The adversary corrupts device $P_{1}$. Assume $A$ can forge a signature in $\text{DistSign}^{\pi}_{A, \Pi}(1^n)$ and $S$ can forge a BLMQ signature in $\text{Sign}_{A, \pi}(1^n)$. Thus, $A$ can use the response of $S$.

1. $S$ receives $(1^n, H_{1}(ID))$, in which $H_{1}(ID)$ is user’s public key.
2. $S$ invokes $A$ and simulates the instructions of $A$ in $\text{DistSign}$ with the input of $(1^n)$ and the output as described in the following:
   (a) If the distributed key generation phase has not completed, then $S$ replies $\bot$ to all queries $(\text{sid}, m)$.
   (b) On receiving a query $(\text{sid}, m)$, if $\text{sid}$ is the first to be queried, $S$ queries the signature in $\text{DistSign}$ and receives $(h, S)$, and $S$ can compute $\mu$ in their BLMQ signature scheme, then $A$ queries $S$ with identifier $\text{sid}$ as below. Case 1:
      i. The first message $m_{1}$ in $(\text{sid}, m_{1})$ as $(\text{prove}, 1, (\mu_{1}, \mu_{2}), (k_{1}, k_{2}))$ that $S$ sends to $\mathcal{F}_{sk}$ if $\mu_{1} = g_{1}^{k_{1}} = g_{2}^{k_{2}}$ and $\mu_{2} = g^{k_{2}}$, then $A$ computes $\{\mu, h'\}$, and replies the message $(\text{prove}, 2, \mu, k_{3})$ to $S$. 


ii. On receiving the message, if the proof is valid, then $S$ sets $s_1 = k_3^{-1} \cdot S$ and replies the message (prove, 3, $s_1$, $k_3^{-1}$) to $A$.

Case 2:

i. $A$ does not execute the instructions of $P_1$ and set $\mu = g^r$ and $h = H_2(m, \mu)$. Then, $A$ sends the message (prove, 1, $\mu$, $x$) to $S$.

ii. On receiving the message, if the proof is valid, $S$ generates random $(s_1, s_2)$ and sends them to $A$.

3. $A$ computes $\sigma^*$ and outputs a signature $(m^*, \sigma^*)$, then $S$ terminals the simulation and outputs $(m^*, \sigma^*)$. □

The view of $A$ in the simulation of key generation phase is different from the real execution of protocol II. The way a honest $P_1$ generates $\mu$ in the real II is that: $P_1$ uses the message from $P_2$ to generate $\mu$, whereas $A$ computes $\mu = g^r$, in which $x$ is a random number. We consider that $S$ behaves as $P_2$ completely in all messages. Since $(x, k_1, k_2, k_3, k_4)$ are all chosen randomly, we determine that there is no distinction between $\mu = g^r$ and $\mu = \mu_1^{-1} \mu_2 g^x$.

In Case I of the distributed signing phase, the simulator $P_2$ generates $s_1$ and $s_2$ using the random numbers $\{k_1, k_2\}$ and its own private key $d_2$. The output of $A$ is computationally indistinguishable to the real output signature in a real protocol. In other words, $A$ outputs a valid signature pair $(m^*, \sigma^*)$ with the same probability in the simulation and in real DistSign. The signatures can be distinguished by verifying whether they match with the public key. When $A$ receives messages from $P_2$, it computes the signature $S^*$. We arrive at the following equation:

$$S^* = k_3 \cdot k_4^{-1} \cdot S = (k_3 k_4^{-1} \cdot k_2 + k_4 + h) \cdot D_{ID} = (k_3 k_4^{-1} \cdot k_2 + k_4 + h) \cdot d_1 d_2 \cdot Q_1.$$  

If $A$ knows the valid signature, then $A$ can compute $d_2$ with a non-negligible probability $\epsilon$. That means $A$ can solve the DL problem with a non-negligible probability $\epsilon$. According to the DL problem, $Pr[d_2|g^{d_1 d_2} \cdot (H_1(ID)+s)] \leq \mu(n)$, which contradicts

$$\epsilon \leq Pr[d_2|g^{d_1 d_2} \cdot (H_1(ID)+s)] \leq \mu(n).$$

Thus, $A$ cannot obtain the private key of $P_2$ even when $P_1$ corrupted and knows the correct signature.

In Case 2 of the distributed signing phase, since the simulator’s private key $d_2$ and KGC’s secret key $s$ are unknown to $A$, $A$ selects a random number $d_3^*$ to construct signature $S^* = (x + h) \cdot d_1 d_2 \cdot Q_1$. There exists:

$$\mu^* = \frac{e(S^*, H_1(ID)Q_2 + P_{pub})}{g^h} = \frac{e(S^*, H_1(ID)Q_2 + sQ_2)}{g^h} = \frac{e(S^*, (H_1(ID) + s)Q_2)}{g^h} = \frac{e((x + h) d_1 d_2 Q_1, (H_1(ID) + s)Q_2)}{g^h} = \frac{g^{(x + h) d_1 d_2 (H_1(ID) + s)}}{g^h} = g^{d_1 d_2 (H_1(ID) + s)}.$$ 

$$\mu = \frac{g^{d_1 d_2 (H_1(ID) + s)}}{g^s} = g^{d_1 d_2 (H_1(ID) + s)}.\$$

$A$ obtains $g^{d_1 d_2^2 (H_1(ID)+s)}$ and outputs $d_2^*$ as the answer to the CDH problem. Suppose that $A$ can select the right $d_2^*$ to make the above equations hold in a non-negligible probability $\epsilon$. That means $A$ can solve the CDH problem in a non-negligible probability $\epsilon$. According to the CDH problem, $Pr[d_2^*|g^{d_1 d_2^2} \cdot (H_1(ID)+s)] \leq \mu(n)$, This contradicts

$$\epsilon \leq Pr[d_2^*|g^{d_1 d_2^2} \cdot (H_1(ID)+s)] \leq \mu(n).$$

Thus, the signature cannot be verified based on the unknown $d_2$ and $s$.

Proof of $b = 2$. The adversary corrupts device $P_2$, and we follow the same steps as the case of $b = 1$. Unlike with $b = 1$, in this case, the signature is computed by $S$ and replied to $A$. In other words, $A$ can forge a signature in DistSign’s $\text{DistSign}_{\text{Ad}1}(1^k)$ and $S$ can forge a BLMQ signature in $\text{Sign}_{\text{Ad}1}(1^k)$. Thus, the adversary $A$ cannot forge a legitimate signature under the circumstances that $S$ private key is not known to $A$. □

This simulation is similar to the above with a difference that the last message from $P_2$ to $P_1$ composes of random numbers. $P_1$ cannot determine whether the random numbers used in messages from $P_2$ is consistent. In this simulation, $S$ computes the signature pair and replies them to $A$. If $A$ can forge a legitimate signature $\sigma^*$, then $A$ can carry out the attack without knowing the private key of $P_1$. We solve this problem by getting $S$ to abort the simulation at some random points. $S$ chooses a random number $i \in \{1, ..., p(n) + 1\}$, in which $p(n)$ means the number of queries. If $S$ chooses correctly, then the simulation is successful.

1. $S$ receives $(i^*, H_1(ID))$, in which $H_1(ID)$ is user’s public key.

2. Let $p(\cdot)$ denotes the number of queries that $A$ executes protocol II. Thus, $S$ can choose a random number $i \in \{1, ..., p(\cdot) + 1\}$.

3. $S$ invokes $A$ and simulates the instructions of $A$ in DistSign with the input of $(1^n)$. The output is described in the following:

(a) If the distributed key generation phase has not completed, then $S$ replies $\perp$ to all queries $(sid, m)$. $P_4$

(b) On receiving a query $(sid, m)$, if $sid$ is the first query, then $S$ computes and sends the corresponding reply (proof, $sid$) to $A$. $P_5$

(c) After $S$ queries the signature in DistSign and returns $(h, S)$, $S$ can compute the $\mu$ in BLMQ signature scheme. Then, $A$ queries $S$ with identifier $sid$ as below:

i. $A$ sends the first message $m_1 = (\text{prove}, 1, \mu_1, \mu_2, (k_1, k_2))$ in $(sid, m_1)$. If $\mu_1 = g_1^{k_1}$ and $\mu_2 = g_2^{k_2}$, then $S$ sets $\mu = g_1^{d_1 k_1 + k_2}$ and $h = H_2(m, \mu)$, and replies the message $(\text{proof}, 2, h)$ to $A$. Otherwise, $S$ terminates the simulation.

ii. The second message $m_2$ in $(sid, m_2)$ is denoted as $(\text{prove}, 3, (s_1, s_2), d_2)$. If this is the $i$ - $th$ query of $A$ to $P_2$, then $S$ terminates the simulation and returns $\perp$. Otherwise, it continues.

4. Whenever $A$ stops the query and outputs the $(m^*, \sigma^*)$, $S$ terminates the simulation and outputs $(m^*, \sigma^*)$. $P_6$
Proof. Suppose $A$ obtains $g^{k_3d_1^{-1}+k_2}$, $i \in \{1, \ldots, p(n)+1\}$ and outputs $k_3d_1^{-1}$ as the answer to k-CAA problem. Suppose that $A$ can compute $k_3d_1^{-1}$ to make the above equations hold in a non-negligible probability $\epsilon$. According to the Collusion Attack Algorithm with k Traitor, $Pr[k_1k_3d_1^{-1}|g^{k_1k_3d_1^{-1}+k_2}] \leq \mu(n)$. This contradicts $\epsilon \leq Pr[k_1k_3d_1^{-1}|g^{k_1k_3d_1^{-1}+k_2}] \leq \mu(n)$.

Thus, $A$ cannot know any information about $P_1$ in the event that $P_2$ is corrupted.

**Theorem 2.** Suppose a P.P.T adversary $A$ is able to specify which signature record belongs to the signature pair in a negligible function $\mu$ for every $n$, then we can say our protocol is blind.

**Proof.** Suppose $A$ can distinguish the signature records with the probability $\epsilon$. In this proof, we separately prove the blindness of device $P_2$ and an external adversary.

**For device $P_2$.** For every signature pair $(m, \sigma)$ generated by device $P_1$, the signature record for $P_2$ includes $\{\mu_1, \mu_2, h', s_i, s_2, k_1, k_2, d_2\}$. Given two signature records $\{\mu_1', \mu_2', h', s'_1, s'_2, k_1', k_2', d_2\}$ and $\{\mu_1'', \mu_2'', h'', s''_1, s''_2, k_1'', k_2'', d_2\}$, and a signature pair $(m, h, S)$. From the signature pair $(m, h, S)$, $P_2$ can compute:

$$\mu = e(S, H_1(ID)Q_2 + P_{pub})$$
$$= g^{k_1k_3d_1^{-1}+k_2+k_4}.$$  

$P_2$ can compute $k_1$ on the basis of $h = h + k_4$. If $P_2$ can specify which of $\{k_1', k_2'\}$ and $\{k_1'', k_2''\}$ belongs to $\mu$, that means, $P_2$ can solve DL problem in a non-negligible probability $\epsilon$. According to Discrete Logarithm Problem, $Pr[k_3, d_1|g^{k_1k_3d_1^{-1}+k_2}] \leq \mu(n)$, it’s contradictory

$$\epsilon \leq Pr[k_3, d_1|g^{k_1k_3d_1^{-1}+k_2}] \leq \mu(n).$$

$P_2$ cannot specify which record belongs to the signature pair.

**For external adversary.** For every signature pair $(m, \sigma)$ generated by device $P_1$, the signature record for an external adversary includes $\{\mu_1, \mu_2, h', s_1, s_2\}$. Given two signature records $\{\mu_1', \mu_2', h', s_1', s_2'\}$ and $\{\mu_1'', \mu_2'', h'', s_1'', s_2''\}$, and a signature pair $(m, h, S)$. From the signature pair $(m, h, S)$, the adversary can compute:

$$\mu = e(S, H_1(ID)Q_2 + P_{pub})$$
$$= g^{k_1k_3d_1^{-1}+k_2+k_4}.$$  

The adversary can compute $k_3$ on the basis of $h = h + k_4$. For $\{\mu_1', \mu_2'\}$, the adversary cannot compute $k_1'$ and $k_2'$ without solving the DL problem in a non-negligible probability $\epsilon$. Thus, the adversary cannot specify which record belongs to the signature pair.  

**6 PERFORMANCE EVALUATION**

We implemented our protocol using the MIRACL library [35] on two Android devices (i.e., Samsung Galaxy S5 with a Quad-core 2.45 G processor, 2G bytes memory and the Google Android 4.4.2 operating system and Google Nexus 6 with a Quad-core, 2.7 GHz processor, 3G bytes memory and the Google Android 7.1.2 operating system) and a PC (Dell with an i7-6700 3.40 GHz processor, 8G bytes memory and the window 10 operating system). In the evaluation, the PC is the KGC which generates partial private keys for both devices. The two devices interact with each other via Bluetooth, as shown in Fig. 4.

The security levels of the different curves are presented in Table 1, and we know that:

1. The security of MNT $k = 6$ curve corresponds to 80 bits cipher length of AES.
2. The security of BN $k = 12$ curve corresponds to 128 bits cipher length of AES.
3. The security of KSS $k = 18$ curve corresponds to 192 bits cipher length of AES.
4. The security of BLS $k = 24$ curve corresponds to 256 bits cipher length of AES.

The implementation consists of three parts, namely: setup, extract, signature and verify. The protocol $\Pi$ 

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**Fig. 4. Model of key generation.**

- $A$ can obtain $p(n)$ correct signatures since $i \in \{1, \ldots, p(n)+1\}$. Thus, we have the following equation:

$$\mu^i = e(S', H_1(ID)Q_2 + P_{pub}) = e(S', (H_1(ID) + s)Q_2)$$
$$= e((k_1k_3d_1^{-1} + k_2 + h)d_1d_2Q_1, (H_1(ID) + s)Q_2)$$
$$= g^{k_1k_3d_1^{-1}+k_2+h} = g^{k_1k_3d_1^{-1}+k_2}.$$  

In this simulation, $A$ keeps its own random number invariant during all queries, and $S$ generates different randomly in every query. Thus, for $p(n)$ correct signatures, $A$ computes $g^{k_2}$ at the beginning of protocol II, and there exists:

$$\mu_1^i = g^{k_1k_3d_1^{-1}+k_2}.$$  

$\mu_2^i = g^{k_1k_3d_1^{-1}+k_2}.$  

$\mu_3^i = g^{k_1k_3d_1^{-1}+k_2}.$  

- $\epsilon \leq Pr[k_3, d_1|g^{k_1k_3d_1^{-1}+k_2}] \leq \mu(n)$.

Thus, $A$ cannot know any information about $P_1$ in the event that $P_2$ is corrupted.
IEEE Proof
627 generates a public parameter set \( P \) and the user’s private key in the setup phase and the extract phase, respectively. \( P \) verifies the validity of the signature after the signature phase. The runtime of each part is shown in Table 2 and Fig. 5.

We analyze the computation cost of the signature phase, whose runtime is displayed in Table 3 and Fig. 6. In the signature phase, step1 refers to the execution of instructions by \( P_1 \) after receiving a request from \( P_2 \), step2 refers to the execution of instructions by \( P_1 \) before sending messages to \( P_2 \), step3 refers to the execution of instructions by \( P_2 \), and step4 refers to the execution of instructions by \( P_1 \) after receiving messages from \( P_2 \).

According to the above evaluation, we analyze the communication cost of BN curve, therefore the length of elements in \( Z_q \) is 256 bits. In the key generation phase of our protocol, the length of the message that the KGC sends to \( P_1 \) is 512 bits, the length of the message that the KGC sends to \( P_2 \) is 512 bits. In the distributed signature generation phase, the length of \( P_1 \)’s first message and second message are 1024 bits and 512 bits respectively. The length of \( P_1 \)’s message is 256 bits. Thus, one can observe that our protocol is efficient and practical for real-world deployment.

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### TABLE 1

<table>
<thead>
<tr>
<th>Security Level</th>
<th>Symmetric cipher key length (AES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MNT ( k = 6 )</td>
<td>80</td>
</tr>
<tr>
<td>BN ( k = 12 )</td>
<td>128</td>
</tr>
<tr>
<td>KSS ( k = 18 )</td>
<td>192</td>
</tr>
<tr>
<td>BLS ( k = 24 )</td>
<td>256</td>
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### TABLE 2

<table>
<thead>
<tr>
<th>Distributed Key Generation and Verification</th>
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<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>( k )</td>
</tr>
<tr>
<td>Setup</td>
</tr>
<tr>
<td>Distributed</td>
</tr>
<tr>
<td>Verify</td>
</tr>
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</table>

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### TABLE 3

<table>
<thead>
<tr>
<th>Running Times of Signature Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>MNT ( k = 6 )</td>
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<td>BN ( k = 12 )</td>
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<tr>
<td>KSS ( k = 18 )</td>
</tr>
<tr>
<td>BLS ( k = 24 )</td>
</tr>
</tbody>
</table>

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Fig. 5. Runtime of the basic operation.

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Fig. 6. Runtime of distributed signing.

### 7 CONCLUSION

In this paper, we proposed a novel, secure and efficient two-party distributed signature protocol, which builds on the identity-based signature scheme in the IEEE P1363 standard. Specifically, a valid signature can be generated without requiring the entire private key to be reconstructed, and the key cannot be generated from one (lost or stolen) device. This can be extremely attractive to mobile device users, as more users own more than one mobile device. In other words, a misplaced or stolen device will not result in the complete compromise of the user’s data. We also proved the security of the protocol, and evaluated its performance on two Android devices.

Future work includes designing an app based on the proposed protocol, and recruiting participants to install the app for more extensive evaluation. Another potential research direction is to implement the app on a broader range of mobile and Internet of Things (IoT) devices (e.g., wearable devices such as smart uniforms and smart weapons) for further evaluation.

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