

# Two Birds with One Stone: Two-Factor Authentication with Security Beyond Conventional Bound

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**Abstract**—As the most prevailing two-factor authentication mechanism, smart-card-based password authentication has been a subject of intensive research in the past two decades, and hundreds of this type of schemes have wave upon wave been proposed. In most of these studies, there is no comprehensive and systematical metric available for schemes to be assessed objectively, and the authors present new schemes with assertions of the superior aspects over previous ones, while overlooking dimensions on which their schemes fare poorly. Unsurprisingly, most of them are far from satisfactory – either are found short of important security goals or lack of critical properties, especially being stuck with the security-usability tension.

To overcome this issue, in this work we first explicitly define a security model that can accurately capture the practical capabilities of an adversary and then suggest a broad set of twelve properties framed as a systematic methodology for comparative evaluation, allowing schemes to be rated across a common spectrum. As our main contribution, a new scheme is advanced to resolve the various issues arising from user corruption and server compromise, and it is formally proved secure under the harshest adversary model so far. In particular, by integrating “honeywords”, traditionally the purview of system security, with a “fuzzy-verifier”, our scheme hits “two birds”: it not only eliminates the long-standing security-usability conflict that is considered intractable in the literature, but also achieves security guarantees beyond the conventional optimal security bound.

**Index Terms**—Two-factor authentication, Smart card loss attack, Zipf’s law, Provable security.

## I. INTRODUCTION

With the rapid development of distributed systems and the increasing demand for sharing services and resources, secure and efficient communications between the distributed user terminals and service providers are raising more and more concerns, and it is of utter importance to protect the systems (e.g., e-commerce systems [1] and cloud systems [2]) and the users’ privacy and security from malicious adversaries. Accordingly, user authentication becomes an essential security mechanism for the application systems to assure one communicating party of the authenticity of the corresponding party by acquisition of corroborative evidence. Among the numerous methods for user authentication, password authentication is the most widely used and acceptable mechanism because of its easy-operation, scalability, compatibility and low-cost advantages [3], [4]. In such password-only authentication schemes (some notable ones include SRP [5], KOY [6] and J-PAKE [7]), each user is assumed to hold a memorable, low-entropy password, while the authentication server needs to store a password-related verifier table necessary to verify the authenticity of users.

An inherent limitation of the traditional password-only mechanism is that, the server has to *store a sensitive verifier*

table that contains the passwords (or passwords in salted hash) of all the registered users. Once the authentication server is compromised, all the users’ passwords will be exposed. Due to human-beings’ inherently limited memory and security budget, the distribution of user-chosen passwords are highly skewed [8]. As a result, even if passwords are stored in salted-hash, this poses no real obstacle for an attacker to recover them by an *overwhelming* percentage (see [9]) by using modern probabilistic cracking techniques [10] and common hardware like GPUs. At Password’12, Gosney [11] showed that a rig of 25 GPUs can test up to 350 billion guesses per second in an offline dictionary attack against traditional hash functions (e.g., NTLM and MD5). More sophisticated password hash functions (e.g., bcrypt and PBKDF2) only provide some relief [12], but with the cost of an honest server increasing by the same factor with the attacker, while the attacker is likely to be better equipped with dedicated password-cracking hardware.

These days it is no news to hear that millions of user accounts are breached in an on-line hacking incident. Some quite recent password data breaches include Adobe (150 million), Evernote (50 million), Anthem (40 million) [13], Rockyou (32 million) [14], Tianya (30 million), Dodonew (16 million), 000webhost (15 million) [15], CSDN (6.4 million) [16], Gmail (4.9 million) and Phpbbs (255K), just to name a few. Some services (e.g., Anthem [13] and Phpbbs [17]) even have been breached more than once during the last five years. What makes things worse is that, users tend to utilize the same password (or slight variations) to access multiple servers (e.g., 43-51% of users as reported in [18]), a compromise of one server will lead to the failure of all other servers, which is described as the “domino effect” of password re-use.

To address the issue of password leakage from a compromised server, threshold password-only authentication schemes (e.g., [19]) have recently been proposed. In such schemes, the password files and user data are distributed over multiple servers, and thus no coalition of servers up to a certain threshold can learn anything about the password. However, they are inherently unable to cope with another emerging issue – password leakage at the user side (e.g., hidden camera, key-loggers and phishing). In order to prevent user side password leakage, leakage-resilient password systems (LRPS) [20] have been advocated. In such schemes, a user needs to input the password indirectly and this imposes an extra burden on common users. Recently, it has been revealed [21] that, to attain both reasonable security and acceptable usability, LRPS schemes have to employ certain trusted devices to well address the security threat of password leakage on user side.

The limitations of threshold and LRPS schemes entrench the third approach – introduce smart cards as “a second line of defense”. This gives rise to the smart-card-based password authentication, often termed as “two-factor authentication”. This kind of authentication was proposed as early as about thirty

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Fig. 1. Smart-card-based password authentication.

years ago [22], and it has been widely deployed for various kinds of security-critical applications, such as e-commerce, e-banking and e-health. It also constitutes the basis of three-factor authentication [23], [24]. The participants of this sort of authentication (see Fig. 1) mainly involve a client  $U$  and an authentication server  $S$  [25], [26]. At first, the user  $U$  registers to  $S$  by submitting her self-chosen personal data (e.g., her identity and password) to  $S$ , then  $S$  securely issues  $U$  a smart-card with some security parameters. This phase is called the user registration phase. Later on,  $U$  and  $S$  authenticate themselves to each other through the login phase. Besides,  $U$  may regularly change her password via the password change phase. Note that, protocols that leverage short message service (SMS) as the second authentication channel and need to cooperate with the telecommunication service provider are out of the scope of this paper.

The most essential security goal of smart-card-based password authentication schemes is to achieve “truly two-factor security” [25], which means that only the user who is in possession of both a smart card and the corresponding password can login the service server. That’s to say, a truly two-factor scheme shall be able to satisfy the following requirements: (1) an attacker in possession of a user’s smart card (and able to extract the contents of the card) should not be able to perform an off-line dictionary attack to recover the user’s password or impersonate the user; and (2) an attacker who learns a user’s password, but does not get this user’s smart card, should not be able to impersonate the user. Besides two-factor security, a practical scheme should also be able to withstand various passive and active attacks [27], [28], such as stolen-verifier attack, denial of service attack, reflection attack and parallel session attack. In addition, it is desirable that schemes can support some important properties [29], [30] like local password change, session key agreement and user anonymity.

There have been hundreds of works dealing with two-factor authentication in recent years (some notable ones include [26], [31]–[34]). However, in most of these studies, the authors present attacks on previous schemes and propose new protocols with assertions of the superior aspects of their schemes, while ignoring the features that their schemes do not attempt (or fail) to provide, thus overlooking dimensions on which their schemes fare poorly. As such, *fair comparison and general consensus are unlikely*. Another common feature of these studies is that, *there is no proper security justification* (let alone an explicit security model) presented, which explains why these protocols previously claimed to be secure turn out to be vulnerable. The research history of this area falls into the unsatisfactory cycle:

NEW PROTOCOL → BROKEN → IMPROVED PROTOCOL →  
BROKEN AGAIN → FURTHER IMPROVED PROTOCOL → ...

For a more concrete grasp, we summarize the “break-fix-break-fix” history of two-factor authentication in Fig. 2. Note that

many other important schemes cannot be incorporated into the figure only because of space constraints.

### A. Motivations

Though considerable efforts have been devoted to the development of secure and efficient two-factor authentication schemes, yet no much progress has been made. This is best illustrated by the recent somewhat surprising revelation [25] that “certain goals are beyond attainment” by just using existing cryptographic design techniques, which means all these previous attempts have failed in vein. A lot of literature has been generated, while little attention has been paid to the *systematic* design and evaluation of this sort of schemes and as a result, there is no common basis that allows schemes to be assessed thoroughly and fairly. As such, more ‘improved schemes’ essentially mean more tangles being added to the already tangled world. This well explains the long-standing failures in getting a secure and efficient two-factor scheme.

As shown in Fig. 2, numerous ‘improvements’ have been proposed, however, most of them have been shortly found either unable to meet some important security goals or short of a few critical features. The crux lies in how to achieve the following two goals simultaneously: (1) truly two-factor security even if the smart cards may be lost and tampered; and (2) local and secure password update. This crux was left as an open problem by Huang et al. in 2014 [27]. Unfortunately, Wang et al. [25] recently find that, under the current two-factor cryptographic protocol design techniques, this problem is highly likely to be intractable. Are there techniques (e.g., ones from the system security domain) beyond the known cryptographic approaches can be used as a hedge against the failure (due to low-entropy passwords and conditional tamper resistance of smart cards) of conventional cryptographic protections?

Another unsatisfactory aspect of the existing literature is that, when evaluating the security guarantees of a password-based scheme (e.g., see some notable schemes in [26], [30], [34], [35]), user-chosen passwords are invariably assumed to be *uniformly* drawn from the password space  $\mathcal{D}$ . Since this assumption is far from realistic, it may give rise to great *misconceptions* of the actual security that a scheme can provide. Under this assumption, even if the parameters in the smart card have been extracted by  $\mathcal{A}$ , the probability of  $\mathcal{A}$ ’s success in one online guessing attempt is precisely  $1/|\mathcal{D}|$ . What a secure two-factor protocol  $\mathcal{P}$  can assure is that, active online guessing is the best that  $\mathcal{A}$  can do and other attack vectors (e.g., offline password guessing, replay and parallel session) are of little help. More specifically, this means  $\mathcal{A}$ ’s optimal advantage (see [26], [30], [34], [36] for example) in attacking  $\mathcal{P}$  is no larger than  $q_{send}/|\mathcal{D}| + \epsilon$ , where  $q_{send}$  denotes the number of online impersonation attempts that  $\mathcal{A}$  engages in and  $\epsilon$  denotes a negligible value. However, user-chosen passwords are far from uniformly distributed and actually, they are on the other extreme: as shown in Sec. V-C, on average  $\mathcal{A}$  would gain an advantage of 3.33%, 4.28%, 8.21%, 15.09% in just 3, 10,  $10^2$  and  $10^3$  online guessing attempts, respectively; but not the assumed advantage of  $3/10^6$ ,  $10/10^6$ ,  $10^2/10^6$ ,  $10^3/10^6$ ,<sup>1</sup> respectively. This misconception about the realistic security guarantee that a scheme can provide is evidently undesirable — *the actual level of security risk of  $\mathcal{P}$  is largely underestimated*.

<sup>1</sup>Note that the size of user password space  $\mathcal{D}$  increases as the user base increases, and thus it is not a constant (see Sec. V-C). It is generally assumed to be about  $2^{20} \approx 10^6$  [37] as a rule of thumb.

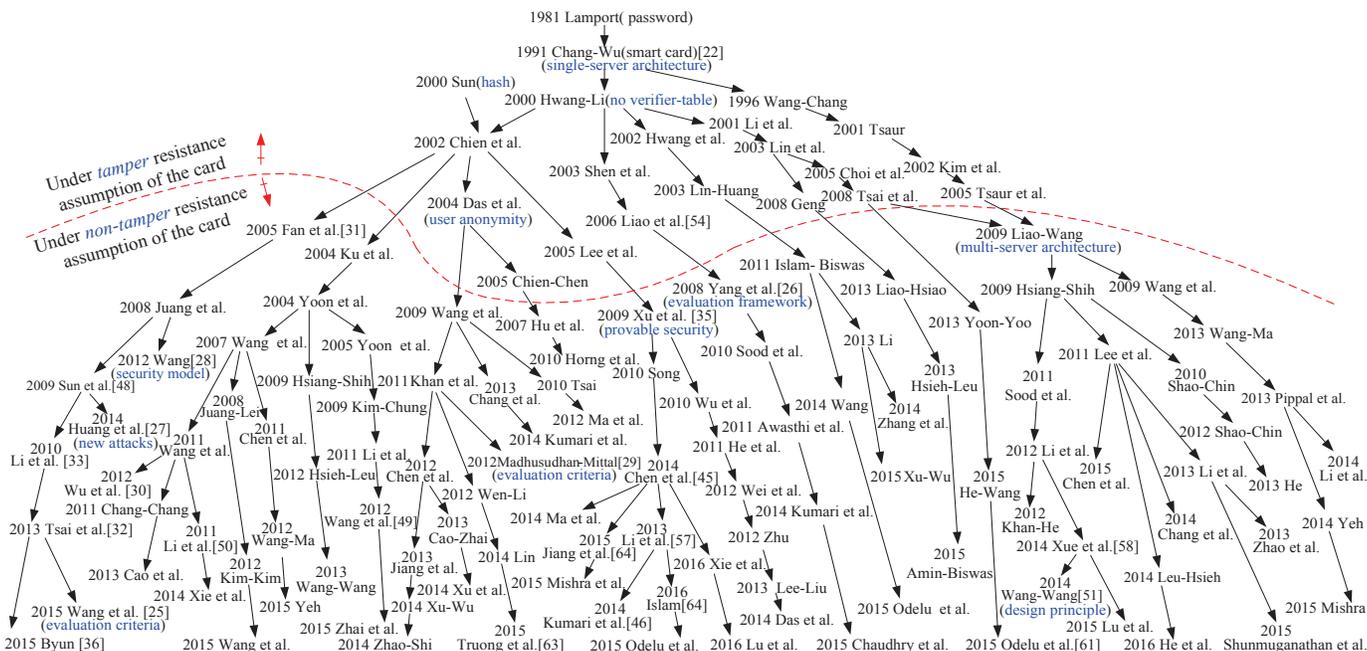


Fig. 2. A brief history of smart-card-based password authentication. Most works follow a similar research pattern: each first proposes practical attacks on scheme(s) in the parent node and then suggests an improved scheme. The seminal contributions of some works are shown in corresponding parentheses.

### B. Our contributions

In this work, we take a first substantial step towards breaking the “break-fix-break-fix” fast knot by investigating into the underlying adversary model and by eliminating the deficiencies (e.g., insufficiencies, ambiguities and redundancies) in the current evaluation criteria set. We explicitly characterize the practical capabilities of an adversary, and suggest a broad set of 12 independent criteria framed as a systematic methodology for comparative evaluation. Though not cast in stone, it is expected that this list of requirements and their specific definitions provide a solid basis to work on. By introducing defensive techniques from the system-security domain, we advance a simple, robust and efficient scheme that addresses the long-standing security-usability conflict and achieves security beyond the conventional optimal security bound  $q_{send}/|\mathcal{D}| + \epsilon$ .

In summary, our contributions are three-fold:

- (1) First, we suggest a systematic framework for evaluating two-factor authentication schemes. It is composed of a practical adversary model as well as a well-refined criteria set. As far as we know, the adversary model is the harshest one to date, and the criterion set is more concrete and comprehensive as compared to related works. The effectiveness and practicality of this framework is demonstrated and tested by rating 67 typical two-factor schemes. It is expected to help facilitate better assessment of current and future schemes.
- (2) Second, we, for the first time, introduce the defensive tactic of “honeywords” [38], traditionally the purview of system security, into two-factor cryptographic protocol design. By integrating “honeywords” with our proposed “fuzzy-verifiers”, our scheme can timely detect user card corruption to thwart online guessing and well addresses the seemingly intractable security-usability problem left in [27]—“whether or not there exist secure smart-card-based password authentication schemes and the password-changing phase does not need any interaction with the server”?

- (3) Third, we demonstrate that the proposed scheme can satisfy all the 12 criteria in our evaluation framework and be formally proved secure in the random oracle model under the harshest adversary model so far. In particular, we use large-scale real-life passwords to show the effectiveness of our integration of “honeywords” with a “fuzzy-verifier”. We also show that our integration technique is generic and can be readily applied to two-factor schemes for various other environments (besides the client-server architecture in this work).

## II. ADVERSARY MODEL AND EVALUATION CRITERIA

To fill the gap in fairly assessing a two-factor authentication scheme, in this Section we explicitly define an adversary model consistent with the reality and present a set of twelve properties framed as a systematic methodology for comparative evaluation. They *together* make it possible for schemes to be rated across a common spectrum. Here we take inspiration from Bonneau et al.’s framework [3] for evaluating web authentication schemes based on usability and security principles.

### A. Adversary model

In the conventional password authenticated key exchange (PAKE) protocols, the attacker  $\mathcal{A}$  is modeled to have full control of the communication channel between the communicating parties [39], [40], such as eavesdropping, intercepting, inserting, deleting, and modifying any transmitted messages over the public channel. To characterize forward secrecy,  $\mathcal{A}$  may also be allowed to corrupt valid parties to attain long-term keys. Besides, previous session key(s) may be obtained by  $\mathcal{A}$  because of a number of reasons such as improper erasure.

Recent studies have reported that, the secret parameters stored in common smart cards could be extracted (or partially extracted) by power analysis attacks [41], [42], the software loophole exploiting attacks (launched on software-supported card, e.g., Java Card) [43] or reverse engineering techniques [44]. Consequently, the leakage of sensitive parameters stored

in the smart card may lead the originally secure schemes vulnerable to the smart card loss problem, such as offline password guessing attack (e.g., the problematic schemes in [32], [45]) and impersonation attack (e.g., the problematic schemes in [35], [46]). Consequently, it is more prudent and desirable to design two-factor schemes under the assumption that the secret keys stored in the smart card could be revealed by some means. What's more, as observed and in-depth investigated by Wang [28] quite recently, malicious card readers also contribute to the security failures of such schemes. Once the card reader is under the control of the attacker (e.g. the card reader is infected with viruses and/or Trojans), the card owner's input password may be intercepted.

However, we restrict the attacker from first intercepting the password via the card reader and then reading the information stored in the card via the stolen (or lost) smart card, otherwise this combination will enable the attacker to trivially break any two-factor authentication protocols. This treatment adheres to "the extreme-adversary principle" [47]: Robust security is to protect against an extremely powerful adversary, of whom the only restricted powers are those that would allow her to trivially break any of this type of schemes. Moreover, this treatment is reasonable in reality: (1) the user is at the scene when she inserts her card into a malicious terminal, and there is little chance for the attacker to launch side-channel attacks (which needs special instruments and attack platforms); (2) the attacker is unlikely to succeed in revealing the sensitive data on the card within a short period of time.

All this implies that the common non-tamper-resistance assumption made about the smart cards shall be *conditional*, i.e., only when the card might be in the attacker's hands for a relatively long time (e.g., the card is lost and picked by the attacker), while in the other scenarios (e.g., the user inserts her card into a malware-infected card reader), the card remains tamper-proof. However, if a memory USB stick is used in such an un-trusted terminal, both the user's password and the data stored in the card memory will be exposed easily without incurring any abnormality. This well explains the essential advantage of using smart cards over employing common cheap memory sticks, even if (conditional) non-tamper resistance assumption of the smart cards are made and smart cards are more expensive than memory sticks.

Our above analysis also invalidates the overly conservative proposition [26], [48] that "we simply consider a smart-card to be a memory card with an embedded micro-processor for performing required operations specified in a scheme." and "we put aside any special security feature that could be supported by the smart card". As memory-card-based schemes are completely insecure when used in un-trusted terminals, all the schemes based on such an extreme assumption like that of [26], [48] are as insecure as memory-card-based schemes, and they can never provide truly two-factor security when used in un-trusted terminals. Therefore, our "conditional" non-tamper resistance assumption of the smart cards is more reasonable than the extreme assumption like that of [26], [48].

Furthermore, for the sake of user-friendliness, a user is often allowed to select her own identity  $ID$  at will (maybe confined to a predefined format) during the registration phase; the user usually tends to choose an easy-to-remember identity which is of low entropy. Thus, user identities can also be

TABLE I. CAPABILITIES OF THE ADVERSARY

C-00	The adversary $\mathcal{A}$ can offline enumerate all the elements in the Cartesian product $\mathcal{D}_{id} \times \mathcal{D}_{pw}$ within a reasonable amount of time, where $\mathcal{D}_{pw}$ and $\mathcal{D}_{id}$ denote the password space and the identity space, respectively.
C-01	The (active) adversary $\mathcal{A}$ has the capability of determining the victim's identity.
C-1	The adversary $\mathcal{A}$ has full control of the communication channel between the communicating parties, such as eavesdropping, intercepting, inserting, deleting, and modifying any transmitted messages over the public channel.
C-2	The adversary $\mathcal{A}$ may either (i) learn the password of a victim via malicious card reader, or (ii) extract the secret data in the lost smart card by side-channel attacks, but cannot achieve both. Otherwise, it is a trivial case.
C-3	The adversary $\mathcal{A}$ can learn the previously established session key(s).
C-4	The adversary $\mathcal{A}$ can learn the server's long-time private key(s) as well as all other stored data only when evaluating the eventual failure of the server.

offline enumerated by  $\mathcal{A}$  within polynomial time. Hence, in practice, it is realistic to assume that  $\mathcal{A}$  can offline enumerate all the  $(ID, PW)$  pairs in the Cartesian product  $\mathcal{D}_{id} * \mathcal{D}_{pw}$  within polynomial time. In contrast, many schemes with user anonymity (e.g. [46], [49]) explicitly assume  $\mathcal{A}$  cannot guess both  $ID$  and  $PW$  correctly at the same time. Such schemes may be problematic under our assumption.

The capabilities of  $\mathcal{A}$  in our model are summarized in Table I. As far as we know, our work, following the works in [26], [28], [50] while providing new insights, is one of the few ones that explicitly specify the capabilities of the adversary. Since a protocol can only be 'secure' under some specific security model, it is hardly able to fairly evaluate the goodness of a scheme if no security model is given. In [28], Wang presented three kinds of security models, namely Type I, II and III. The last model is the most powerful one to date, and it mainly makes three assumptions:

- (1)  $\mathcal{A}$  is allowed to have full control of the communication channel, which is consistent with C-1 in Table I;
- (2) The smart card is assumed to be non-tamper resistant and the user's password may be intercepted by  $\mathcal{A}$  using a malicious card reader, but not both, which is consistent with C-2 in Table I;
- (3) The smart card has no counter protection, i.e.  $\mathcal{A}$  can issue a large amount of queries to the card using a malicious card reader to learn some useful information.

With regard to Assumption 3, we argue that this assumption may not be of much practical significance, because whether it is valid or not in practice has little relevance with protocol security under Assumption 2. On the one hand, if there is no verification of the input password before the run mode of the smart card, the only way that  $\mathcal{A}$  can learn some useful information (except the static data stored in the card, which can be learnt by  $\mathcal{A}$  under Assumption 2) is to interact with the remote server, which can be effectively thwarted by the server, e.g., locking the corresponding user account after a few failed login attempts. On the other hand, if this verification exists,  $\mathcal{A}$  can always find the password that passes the verification by exhaustively inputting her guessing passwords into the malicious card reader (and with Assumption 2, secret data can also be extracted out), which is explicitly not allowed in the Type III model. Hence, Assumption 3 is not considered in our model. As with [49], we may simply assume that there is counter protection in the card, i.e., the card will be locked for a time period if the query number exceeds a certain threshold (e.g., the GSM SIM card V2 or later has this capability).

According to the above analysis, our model is closest to the Type III model in [28], and the key difference is that  $\mathcal{A}$

in Type III model is not provided with C-3 and C-4. Hence, Type III may fail to deal with some important security features, such as forward secrecy and resistance to known key attack. As compared to Li-Lee's model [50] and Yang et al.'s model [26], our model has explicitly taken the malicious card reader into consideration, and  $\mathcal{A}$  is further armed with C-3 and C-4.

Moreover,  $\mathcal{A}$  in our model is assumed to be able to offline enumerate all the  $(ID, PW)$  pairs in the Cartesian product  $\mathcal{D}_{id} * \mathcal{D}_{pw}$  within polynomial time, which enables our model to deal with the special security issues such as resistance to offline password (more precisely,  $(ID, PW)$  pair) guessing attack and undetectable online password guessing attack, in dynamic-ID-based schemes (e.g., [46], [49]). Note that, C-01 has also been yet implicitly made in [26], [28], both of which do not concern the admired feature of user anonymity, for the emphasis on C-01 (e.g., we deliberately separate it from C-00 and list it as an independent item) is meaningful only when user anonymity is considered. All in all, our model is stronger and practically reasonable as it incorporates the previous assumptions as well as other new practical assumptions (i.e., the computational power of  $\mathcal{A}$  is large but not omnipotent), especially when considering the proliferation of mobile device use cases. Particularly, the practicality of our model is confirmed by the fact that, under such a strong model, secure and efficient protocols can still be built (see Sec. IV).

### B. Evaluation criteria

As pointed out by Yang et al. [26], although the construction and security analysis of smart-card-based password authentication schemes have a long history, there is no common set of desirable security properties that has been widely adopted for the construction of this type of schemes. Later on, Madhusudhan and Mittal [29] showed that earlier criteria sets have redundancies and ambiguities, and hence they proposed a new criteria set of nine security goals and ten desirable features. Since the security goals of their criteria are based on the non-tamper resistance assumption of smart cards, their set is superior to other proposed sets. However, it still has some redundancies (as will be discussed later) and also fails to notice some inherent conflicts (see [25]) among the criteria.

Considering these earlier criteria-related studies [25], [26], [29], based on our cryptanalysis experience of 67 typical schemes (see Appendix A in the supplemental file <http://bit.ly/2bDTXNa>) and some of our earlier attacking results [25], [51], [52], and further using the iteration methodology [3] for criterion refinement, we put forward a broad list of 12 independent criteria in terms of user friendliness and security that a two-factor authentication scheme shall satisfy:

- C1. **No password verifier-table:** the server does not need to maintain a database for storing the passwords or some derived values of the passwords of its clients;
- C2. **Password friendly:** the password is memorable, and can be chosen freely and changed locally by the user;
- C3. **No password exposure:** the password cannot be derived by the privileged administrator of the server;
- C4. **No smart card loss attack:** the scheme is free from smart card loss attack, i.e., unauthorized users getting a victim's card should not be able to easily change the password of the smart card, recover the victim's password by using online, offline or hybrid guessing attacks, or impersonate the user to login to the system,

even if the smart card is obtained and/or secret data in the smart card is revealed;

- C5. **Resistance to known attacks:** the scheme can resist various kinds of basic and sophisticated attacks, including offline password guessing attack, replay attack, parallel session attack, de-synchronization attack, s-tolen verifier attack, impersonation attack, key control, unknown key share attack and known key attack;
- C6. **Sound repairability:** the scheme provides smart card revocation with good repairability, i.e., a user can revoke the smart card without changing her identity;
- C7. **Provision of key agreement:** the client and the server can establish a common session key for secure data communications during the authentication process;
- C8. **No clock synchronization:** the scheme is not prone to the problems of clock synchronization and time-delay, i.e., the server needs not to synchronize its time clock with these time clocks of all input devices used by smart cards, and vice versa;
- C9. **Timely typo detection:** the user will be timely notified if she inputs a wrong password by mistake when login;
- C10. **Mutual authentication:** the user and server can verify the authenticity of each other.
- C11. **User anonymity:** the scheme can protect user identity and prevent user activities from being traced;
- C12. **Forward secrecy:** the scheme provides the property of perfect forward secrecy.

It is worth pointing out that the criterion C4 deals with attacking scenarios where  $\mathcal{A}$  has obtained access to the victim's smart card, while C5 deals with scenarios where  $\mathcal{A}$  has *no* access to the victim's smart card. The criterion C4 considers the traditional smart-card-loss issues (see [29]) as well as attacking scenarios newly revealed (e.g., attacks by interactively using the server as an oracle [27] and attacks by returning the extracted card [25]), and see [52] for a taxonomy of eight attacking scenarios. C5 is based on the list of basic attacks [39], [40] that a password-only authentication scheme needs to guard against and on security notions [53] that relate to session keys,<sup>2</sup> as well as on new attack vectors (e.g., stolen verifier attack) that arise in the two-factor authentication environment.

We now show that our criteria set not only eliminates the redundancies and ambiguities of the conventional criteria sets, but also facilitates cryptanalysis due to its concreteness. We first take Madhusudhan and Mittal's set [29], the most recent and representative set ever proposed, as a concrete example of redundancies. Its criteria "SR9. Insider attack" and "G3. No password reveal:" essentially mean the same thing, while its criterion "G4. Password dependent" is completely included in its criterion "SR6. Smart card loss attack", for a scheme which is not password dependent will be prone to smart card loss attack but not to other attack(s). Besides, the important property "free password change", which is widely considered in other sets like [26], [30], is missing in [29]. One can check that the criteria in [29] is entirely included into our set.

Then, we proceed to show the ambiguities of the previous requirement sets. Unlike the criteria set proposed in Liao et al. [54], the criterion concerning with performance, which

<sup>2</sup>Note that, authentication protocols in which the server also serves as the registration center of clients are inherently unable to resist key compromise impersonation (KCI) attack, and therefore the KCI notion is not considered.

says “The scheme must be efficient and practical”, is not incorporated into our set. The main reason is that, this criterion does not seem to be measurable (and thus ambiguous) without referring to other related schemes. In other words, isolating it from the criteria set can make our set more concrete and decidable. Further, the efficiency of a scheme may depend on the implementation environment, while practicality is largely relevant to the target applications. Except this criterion, all the other criteria in [30], [54] are included into our set. Although the criterion related with performance is not listed in Yang et al.’s set [26], their set is merely composed of four criteria (i.e., C2-C5) and evidently too limited to be of practical operability.

Finally, as one could argue that there may be the probability that someone else will claim tomorrow that a list they come up with is better, we acknowledge that our list of criteria is not complete, and indeed, any such list could always be expanded. We resist the temptation to create a new criterion and mainly focus on the criteria that have already been extensively discussed in the past literature, because a criterion that has drawn little or no previous attention probably can not be considered essential. Though not cast in stone, our list is more concrete and comprehensive than related ones and is expected to help facilitate a deeper understanding of the pros and cons of the current and future two-factor schemes. This is of fundamental importance for security engineers to make their choices correctly and for protocol designers to develop practical schemes with better usability-security tradeoffs.

**Summary.** Extensive comparisons show that, our adversary model is the harshest one so far and our criterion set is more concrete, concise, and comprehensive as compared to related works. As any cryptographic protocol meets its goals only within some security model [25], [39], we expect it is the systematic evaluation framework, *as a whole*, that constitutes the main long-term scientific value, but neither our adversary model nor our criteria set alone does. The effectiveness of this framework is demonstrated and tested by rating 67 two-factor schemes without hidden agenda, as summarized in a carefully constructed table in Appendix A. Both the rating criteria and their definitions were iteratively refined and re-categorized when evaluating these 67 schemes. One can see that, each criterion can be *satisfied* by at least 15 schemes and in the meantime, it is also *unmet* by at least 7 schemes. This suggests the *necessity* of each criterion. On the other hand, each scheme fails to fulfill at least one criterion, which implies the comprehensiveness of our list and highlights the need for more efforts to design better schemes.

### III. FORMAL SECURITY MODEL

To formally capture the capabilities of an adversary in smart-card-based password authentication and specify how the adversary interacts with honest parties, we recall the BPR2000 security model [39] where the adversary’s capabilities are modelled through queries and define some security notions. However, we do not use the original BPR2000 model directly, but adopt the reified version proposed by Bresson et al. [55] with a few key modifications so that we can define the special security goals (e.g., security against smart card loss attack) for two-factor authentication. We refer the reader unfamiliar with the BPR2000 model to [39], [55] for more details.

**Players.** In a two-factor protocol  $\mathcal{P}$ , there are two protocol participants involved, namely, a user  $U \in \text{User}$  and a server

$S \in \text{Server}$ , where  $\text{User}$  and  $\text{Server}$  are disjoint. Each of them may have several instances called oracles involved in distinct, possibly concurrent, executions of  $\mathcal{P}$ . We denote client instances and server instances by  $U^i$  and  $S^j, i, j \in \mathbb{Z}$ , and denote any kind of instance by  $I \in \text{User} \cup \text{Server}$ .

**Long-lived keys.** In the registration phase, long-lived keys and public parameters (if any) are established for each participant. The server  $S$  is provided with a pair of long-term public and private keys  $(pub_{key}, pri_{key})$  in a public-key based scheme (or a single symmetric key  $sym_{key}$  in a symmetric-key based scheme), while each user  $U \in \text{User}$  with identity  $ID_U$  is equipped with a password  $PW_U$  which is assumed to be drawn from a Zipf-distributed [8] “dictionary”  $\mathcal{D}$  of small size  $|\mathcal{D}|$ , where  $|\mathcal{D}|$  is a fixed constant which is independent of the system security parameter.<sup>3</sup> The vector  $\langle ID_U, TPW_U \rangle_{U \in \text{User}}$  is kept on  $S$ , where  $TPW_U$  is an injective transformation of  $\langle ID_U, PW_U \rangle$  and  $pri_{key}/sym_{key}$ . Additionally,  $S$  stores some non-sensitive user-specific data as well as a few necessary public parameters into a smart card and issues it to  $U$ .

**Queries.** The interaction between an adversary  $\mathcal{A}$  and the protocol participants occurs only via oracle queries, which model the adversary capabilities in a real attack. The query types available to  $\mathcal{A}$  are defined as follows.

- $\text{Execute}(U^i, S^j)$ : This oracle query is used to model passive (eavesdropping) attacks of the adversary. The output of this query consists of the messages that were exchanged during the honest execution of the protocol.
- $\text{Send}(I, m)$ : This query models an active attack, in which  $\mathcal{A}$  may send a message to instance  $I$  and get back the response that  $I$  generates in processing the message  $m$  according to the protocol  $\mathcal{P}$ . A query  $\text{Send}(U^i, \text{Start})$  initializes the protocol.  $\text{Start}$  is a message, and thus  $\mathcal{A}$  receives the flow that the client should send out to  $S$ .
- $\text{Test}(I)$ : This oracle query is not used to simulate the adversary’s attack, but to define session key’s semantic security. If no session key for instance  $I$  is defined, then undefined symbol  $\perp$  is returned. Otherwise, a private coin  $c$  is flipped. If  $c = 1$  then the session key  $sk$  is returned to  $\mathcal{A}$ , otherwise a random key of the same size is returned. This query can only be directed towards a fresh instance and called only once during its execution.
- $\text{Reveal}(I)$ : This query models the misuse of session keys (i.e. C-3 in Table I). It returns the session key  $sk$  of participant instance  $I$  to the adversary, if the target instance actually “holds” a session key, and  $I$  and its partner were not asked by a  $\text{Test}$  query. Otherwise the  $\perp$  is returned.
- $\text{Corrupt}(I, a)$ : This query models corruption capability of  $\mathcal{A}$  (i.e. C-2 and C-4 in Table I).  $\mathcal{A}$  can break either one of the two authentication factors of clients, but not both:
  - If  $I = U, a = 1$ , it outputs the password  $PW_U$  of  $U$ ;
  - If  $I = U, a = 2$ , it outputs all the security parameters that are stored in the smart card.

<sup>3</sup>When performing security proofs, most existing password-based protocols (e.g., [6], [7], [26], [34], [39]) assume passwords to be uniformly distributed. However, human-chosen passwords are actually extremely skewed (i.e., Zipf-distributed [8]) and as will be shown in Fig. 3 and Sec. V, there are *three to four order-of-magnitude difference* in  $\mathcal{A}$ ’s advantages between these two password distributions. This defeats the purpose of using formal methods to accurately assess  $\mathcal{A}$ ’s advantages. Thus, the uniform assumption of passwords is undesirable and shall be abandoned.

- If  $I = S, a = 1$ , it outputs the private key  $pr_{ikey}$  (or the symmetric key  $pr_{key}$ ) of the server  $S$  and  $\langle ID_U, TPW_U \rangle_{U \in U_{\text{ser}}}$  that are stored in  $S$ 's backend database.

It is not difficult to check that, the above oracle queries indeed can characterize all the adversary's capabilities listed in Table I, and thus our model defined here facilitates capturing various known attacks, such as impersonation, smart card loss, stolen-verifier, offline password guessing, known key, forward secrecy and passive eavesdropping. Here we mainly focus on security notions and as for the formal definition of user anonymity, readers are referred to a follow-up study [56].

**Partnering.** We define *partnering* by using the notion of session identifier  $sid$ . This notion is essential in the later formulation of *Freshness*. Let  $U^i$  and  $S^j$  be a pair of instances. We say that the instances  $U^i$  and  $S^j$  are partnered if the following conditions are true: ① Both instances have accepted; ② Both instances shared the same  $sid$ ; ③ The partner identifier (pid) of  $U^i$  is  $S$  and vice-versa. In general, as with [6], we let  $sid$  be the ordered concatenation of all messages sent and received by the instance  $U^i$  (or  $S^j$ ).

**Freshness.** The *freshness* notion is a key point in the definition of protocol security and captures the intuitive fact that a session key can not be trivially known to the adversary. We say that an instance  $I$  is fresh if: ①  $I$  has accepted and computed a session key; ② Neither  $I$  nor its partner has been asked for a Reveal-query; ③ At most one kind of Corrupt-query is made to  $U$  from the beginning of the game.

**Correctness.** If  $U^i$  and  $S^j$  are partnered and accepted, then they end up with the same session key  $sk_U^i = sk_S^j$ .

**Authentication.** A fundamental goal of the authentication schemes is to prevent the adversary from impersonating the client or the server. We denote by  $\text{Adv}_{\mathcal{P}}^{\text{auth}}(\mathcal{A})$  the probability that  $\mathcal{A}$  successfully impersonates as an instance of either  $U$  or  $S$  in an execution of  $\mathcal{P}$ . This means that  $S$  (resp.  $U$ ) agrees on a key, while it is shared with no instance of  $U$  (resp.  $S$ ).

Now it remains to define what is a secure two-factor authentication protocol. For simplicity, here we do not take into account forward secrecy. According to the security results reported on password-only protocols [6], [40], the *best* strategy that an adversary in a *secure* password-only protocol can adopt is to perform the online password guessing attack (and test one password candidate in each guessing session), which can be relatively easily detected and conquered.

As for two-factor schemes based on the (conditional) non-tamper resistance assumption of smart cards, it is desirable that online password guessing attack shall also be  $\mathcal{A}$ 's best possible strategy to impersonate a party: (1) if the password is compromised but not the smart card,  $\mathcal{A}$  can hardly succeed because there are still high-entropy secrets stored on the card; and (2) if the smart card is compromised but not the password,  $\mathcal{A}$  can at least try one password candidate each time to impersonate the victim by interacting with the server, which is essentially the online guessing attack. Note that, instances with which the adversary interacts via Execute-queries are not counted as on-line attacks. This motivates the following definition: The two-factor protocol  $\mathcal{P}$  is said to achieve mutual authentication if, for any probabilistic polynomial time (PPT) adversary  $\mathcal{A}$  making at most  $q_{\text{send}}$  on-line attacks, there exists a negligible function  $\epsilon(\cdot)$  such that:

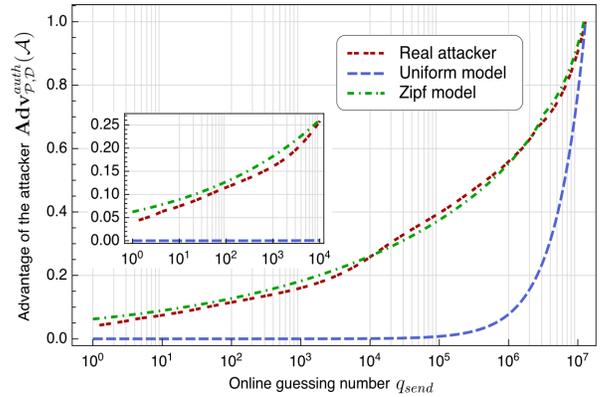


Fig. 3. Online guessing advantages of a real attacker, the uniform-modeled attacker and our Zipf-modeled attacker (using 30.9M Tianya passwords).

$$\text{Adv}_{\mathcal{P},\mathcal{D}}^{\text{auth}}(\mathcal{A}) \leq C' \cdot q_{\text{send}}^{s'} + \epsilon(\ell)$$

where  $\mathcal{D}$  is a password space with its frequency distribution follows a Zipf's law [8],  $C'$  and  $s'$  are the Zipf parameters, and  $\ell$  is the system security parameter. Here we use the Zipf model of Tianya, where  $|\mathcal{D}|=12,898,437$ ,  $C'=0.062239$  and  $s'=0.155478$  [8]. This result requires that calls to the Execute-query (i.e., passive attacks) shall be useless to  $\mathcal{A}$ .

To help gain a better understanding of the soundness of our above Zipf-model-based formulation (i.e.,  $C' \cdot q_{\text{send}}^{s'} + \epsilon$ ) over the existing uniform-model-based one (i.e.,  $q_{\text{send}}/|\mathcal{D}| + \epsilon$ ), readers are referred to some concrete schemes in [26], [30], [34], [36]), we take the Zipf model of Tianya in [8] as an example. Fig. 3 shows the gaps among the *captured* advantages of the real attacker, the uniform-model-based attacker and our Zipf-model-based attacker. As  $|\mathcal{D}|$  is generally assumed to be in millions and  $q_{\text{send}}$  to be in thousands [37], the advantage of a uniform-model-based attacker (i.e.,  $q_{\text{send}}/|\mathcal{D}|$ ) will be *theoretically* less than 1.0%. However, this value for *the real attacker* is about 25.78%. Fortunately, our Zipf attacker well approximates the real attacker, giving an approximation of  $\mathcal{A}$ 's advantage to be 26.06%. Fig. 3 shows the uniform-model always greatly underestimates  $\mathcal{A}$ 's advantage when the guess number  $q_{\text{send}}$  is below  $10^7$ . In contrast, the Zipf-model slightly overestimates  $\mathcal{A}$ 's real advantage when  $q_{\text{send}}$  is below  $10^4$ , when  $q_{\text{send}}$  is over  $10^4$  but below  $10^6$ , our Zipf-model slightly underestimates  $\mathcal{A}$ 's real advantage. This demonstrates that *our Zipf-model-based formulation is particularly suitable for capturing  $\mathcal{A}$ 's online guessing advantage* (i.e., when the guess number is small, e.g.,  $q_{\text{send}} \leq 10^6$ ).

**Semantic security.** Another major concern of authentication schemes with key agreement is to protect the privacy of the session key. In a protocol run of  $\mathcal{P}$ ,  $\mathcal{A}$  can ask a polynomial number of Execute-query, Reveal-query and Send-query. It can also ask a single Test query to a *fresh* instance. At the end of the game,  $\mathcal{A}$  outputs a guess bit  $c'$  for the bit  $c$  involved in the Test-query. We say that  $\mathcal{A}$  wins the game if  $c' = c$ , and this event is denoted by **Succ**. Accordingly, the advantage of  $\mathcal{A}$  in breaking the semantic security of the protocol  $\mathcal{P}$  is defined as

$$\text{Adv}_{\mathcal{P}}^{\text{ake}}(\mathcal{A}) = 2\text{Pr}[\text{Succ}(\mathcal{A})] - 1 = 2\text{Pr}[c' = c] - 1$$

where the probability space is over all the random coins of the adversary and all the oracles.

The discussions about the notion of "Authentication" also motivate the following definition: The two-factor protocol  $\mathcal{P}$

is said to be semantically secure if, for any probabilistic polynomial time adversary  $\mathcal{A}$  making at most  $q_{send}$  on-line attacks, there exists a negligible function  $\epsilon(\cdot)$  such that:

$$\text{Adv}_{\mathcal{P}, \mathcal{D}}^{ake}(\mathcal{A}) \leq C' \cdot q_{send}^{s'} + \epsilon(\ell)$$

where the parameters are the same with those of the definition of ‘‘Authentication’’. This result requires that calls to the Execute-query (i.e., passive eavesdropping) are useless to  $\mathcal{A}$ .

#### IV. OUR PROPOSED SCHEME

In this section, we present a simple, robust yet efficient smart-card-based password authentication scheme that is able to provide all of the twelve criteria introduced in Sec. II-B. Our scheme consists of four phases: the registration phase, the login phase, the verification phase and the password change phase. For ease of presentation, we employ some intuitive abbreviations and notations listed in Table II.

TABLE II. NOTATIONS AND ABBREVIATIONS

Symbol	Description	Symbol	Description
$U_i$	$i^{th}$ user	$x$	the secret key of $S$
$S$	remote server	$\oplus$	the bitwise XOR operation
$\mathcal{A}$	the adversary	$\parallel$	string concatenation
$ID_i$	identity of user $U_i$	$\Rightarrow$	a secure channel
$PW_i$	password of user $U_i$	$\rightarrow$	a common channel

##### A. Registration phase

The protocol is defined over a finite cyclic group  $\mathbb{G} = \langle g \rangle$  of order a  $\ell$ -bit prime number  $q$ . This group could be a finite group, or it could be an elliptic curve group. In this paper, we assume  $\mathbb{G}$  is a prime order subgroup of  $\mathbb{Z}_p^*$ , where  $\mathbb{Z}_p^* = 1, 2, \dots, p-1$  and  $p$  also is a large prime number such that  $q|p-1$ . Hash functions from  $\{0, 1\}^* \rightarrow \{0, 1\}^{l_i}$  are denoted by  $\mathcal{H}_i(\cdot)$ , where  $l_i$  is the bit length of function output (e.g.  $l_i = 160$ ) and  $i = 0, 1, 2, 3$ . We also define a medium integer  $n_0$ ,  $2^4 \leq n_0 \leq 2^8$ , which determines the capacity of the pool of the  $(ID, PW)$  pair against online guessing (and relates, as we will show later, to the fuzzy-verifier  $A_i$ ). Let  $(x, y = g^x \text{ mod } p)$  denote the server  $S$ 's private key and its corresponding public key, where  $x$  is kept secret by  $S$  and  $y$  is stored inside each user's smart card. The registration phase performs as follows:

- Step R1.  $U_i$  chooses her identity  $ID_i$ , password  $PW_i$  and a random string  $b$ .
- Step R2.  $U_i \Rightarrow S : \{ID_i, \mathcal{H}_0(b||PW_i)\}$ .
- Step R3. On receiving the registration message from  $U_i$  at time  $T$ ,  $S$  first picks a random number  $a_i$  and computes  $A_i = \mathcal{H}_0((\mathcal{H}_0(ID_i) \oplus \mathcal{H}_0(b||PW_i)) \text{ mod } n_0)$ . Then  $S$  checks whether  $U_i$  is a registered user. If it is  $U_i$ 's initial registration,  $S$  creates a new entry for  $U_i$  in the account-database and stores  $\{ID_i, T_{reg}=T, a_i, \text{Honey\_List}=\text{NULL}\}$  in this entry. Otherwise,  $S$  only updates the values of  $T_{reg}$  to  $T$ ,  $a_i$  to newly created  $a_i$ , and Honey\_List to NULL in the existing entry for  $U_i$ . Next,  $S$  computes  $N_i = \mathcal{H}_0(b||PW_i) \oplus \mathcal{H}_0(x||ID_i||T_{reg})$ .
- Step R4.  $S \Rightarrow U_i$ : A smart card with security parameters  $\{N_i, A_i, A_i \oplus a_i, q, g, y, n_0, \mathcal{H}_0(\cdot), \dots, \mathcal{H}_3(\cdot)\}$ .
- Step R5. Upon receiving the smart card  $SC$ ,  $U_i$  activates it. Then  $SC$  requests  $U_i$  to enter the random string  $b$  twice to confirm its correctness.

Note that, in Step R1 the user  $U_i$  may write the random string  $b$  on a piece of paper when choosing  $b$ , and this paper

can be torn once  $b$  has been entered into the smart card after the completion of Step R5. In this way, there is no need for  $U_i$  to remember  $b$  at any time, and thus  $b$  can be selected by  $U_i$  as random (long and unpredictable) as possible to attain C3: no password exposure (see Sec. II-B). In addition, we assume, for simplicity, that the record  $T_{reg}$  is secure enough (e.g. 128 bits) against brute-force guessing. In some contexts,  $T$  may be only 64-bits long or even shorter, in this case we can set  $T_{reg} = T \parallel X$ , where  $X$  is a large random number.

##### B. Login phase

This phase involves the following operations:

- Step L1.  $U_i$  inserts her smart card  $SC$  into the card reader and inputs  $ID_i^*, PW_i^*$ .
- Step L2.  $SC$  computes  $A_i^* = \mathcal{H}_0((\mathcal{H}_0(ID_i^*) \oplus \mathcal{H}_0(b||PW_i^*)) \text{ mod } n_0)$  and verifies the validity of  $ID_i^*$  and  $PW_i^*$  by checking whether  $A_i^*$  equals the stored  $A_i$ . If they are not equal, the session is terminated.
- Step L3.  $SC$  chooses a random number  $u$  and computes  $C_1 = g^u \text{ mod } p$ ,  $Y_1 = y^u \text{ mod } p$ ,  $k = \mathcal{H}_0(x||ID_i||T_{reg}) = N_i \oplus \mathcal{H}_0(b||PW_i^*)$ ,  $a_i = (A_i \oplus a_i) \oplus A_i$ ,  $CID_i = ID_i^* \oplus \mathcal{H}_0(C_1||Y_1)$ ,  $CAK_i = (a_i||k) \oplus \mathcal{H}_0(Y_1||C_1)$  and  $M_i = \mathcal{H}_0(Y_1||k||CID_i||CAK_i)$ .
- Step L4.  $U_i \rightarrow S : \{C_1, CID_i, CAK_i, M_i\}$ .

Note that, the input to  $\mathcal{H}_0(\cdot)$  in the formulation of  $CAK_i$  is reversed as compared to that of  $\mathcal{H}_0(\cdot)$  in  $CID_i$ . This aims to prevent information leakage — otherwise,  $\mathcal{A}$  can recover  $a_i||k$  by computing  $CID_i \oplus CAK_i$ . In addition, the verifier  $M_i$  in the login request is added to cope with the ability C-01 of the adversary, i.e., to resist against a new kind of smart-card loss attack uncovered by Huang et al. in 2014 [27].

##### C. Verification phase

After receiving the login request  $\{C_1, CID_i, CAK_i, M_i\}$ , the server  $S$  performs the following operations:

- Step V1.  $S$  computes  $Y_1 = (C_1)^x \text{ mod } p$  using its private key  $x$ . Then,  $S$  derives  $ID_i = CID_i \oplus \mathcal{H}_0(C_1||Y_1)$  and checks whether  $ID_i$  is in the correct format. If  $ID_i$  is not valid, the session is terminated. Otherwise,  $S$  proceeds to the next step.
- Step V2.  $S$  computes  $k = \mathcal{H}_0(x||ID_i||T_{reg})$  and  $M_i^* = \mathcal{H}_0(Y_1||k||CID_i||CAK_i)$ , where  $T_{reg}$  is extracted from the entry corresponding to  $ID_i$  in the account-database. If  $M_i^* \neq M_i$ , the session is terminated.
- Step V3.  $S$  derives  $a_i' || k' = CAK_i \oplus \mathcal{H}_0(Y_1||C_1)$ , and checks whether  $a_i'$  equals the stored  $a_i$ . An equality implies a login request with the right  $A_i$ .  $S$  rejects if they are not equal. Otherwise,  $S$  checks whether the derived  $k'$  equals the computed  $k$ . If they are equal,  $S$  proceeds to the next step. If they are unequal,  $S$  now knows that  $a_i' = a_i$  but  $k' \neq k$ , implying that there is a  $1 - \frac{1}{2^{n_0}}$  probability that  $U_i$ 's card has been corrupted. Accordingly,  $S$  performs either (1) inserts  $k'$  into Honey\_List when there are less than  $m_0$  (e.g.,  $m_0 = 10$ ) items in Honey\_List; or (2) suspends  $U_i$ 's card (i.e., when there are  $m_0$  items in Honey\_List) until  $U_i$  re-registers.
- Step V4.  $S$  generates a random number  $v$  and computes the temporary key  $K_S = (C_1)^v \text{ mod } p$ ,  $C_2 = g^v \text{ mod } p$  and  $C_3 = \mathcal{H}_1(ID_i||ID_S||Y_1||C_2||k||K_S)$ .
- Step V5.  $S \rightarrow U_i : \{C_2, C_3\}$ .

- Step V6. On receiving the reply message from the server  $S$ , the smart card computes  $K_U = (C_2)^u \bmod p$ ,  $C_3^* = \mathcal{H}_1(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_U)$ , and compares  $C_3^*$  with the received  $C_3$ . This equivalency authenticates the legitimacy of the server  $S$ , and  $U_i$  goes on to compute  $C_4 = \mathcal{H}_2(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_U)$ .
- Step V7.  $U_i \rightarrow S : \{C_4\}$
- Step V8. Upon receiving  $\{C_4\}$  from  $U_i$ ,  $S$  first computes  $C_4^* = \mathcal{H}_2(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_S)$  and then checks if  $C_4^*$  equals the received  $C_4$ . If this verification holds,  $S$  authenticates the user  $U_i$  and the login request is accepted else the connection is terminated.
- Step V9. The user  $U_i$  and the server  $S$  agree on the common session key  $sk_U = \mathcal{H}_3(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_U) = \mathcal{H}_3(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_S) = sk_S$  for securing future data communications.

#### D. Password change phase

For the sake of security, user friendliness and communication efficiency (i.e., to satisfy the criterion C2), this phase is performed locally without the hassle of interaction with the remote server, and it involves the following steps:

- Step P1.  $U_i$  inserts her smart card into the card reader and inputs  $ID_i$  and the original password  $PW_i$ .
- Step P2. The card computes  $A_i^* = \mathcal{H}_0((\mathcal{H}_0(ID_i) \oplus \mathcal{H}_0(b \| PW_i)) \bmod n_0)$  and verifies the validity of  $A_i^*$  by checking whether  $A_i^*$  equals to the stored  $A_i$ . If the verification holds, it implies the input  $ID_i$  and  $PW_i$  are valid with a probability of  $\frac{n_0-1}{n_0} (\approx \frac{99.61}{100}$ , when  $n_0=2^8$ ). Otherwise, the smart card rejects.
- Step P3. The smart card asks the cardholder to resubmit a new password  $PW_i^{new}$  and computes  $N_i^{new} = N_i \oplus \mathcal{H}_0(b \| PW_i) \oplus \mathcal{H}_0(b \| PW_i^{new})$ ,  $A_i^{new} = \mathcal{H}_0((\mathcal{H}_0(ID_i) \oplus \mathcal{H}_0(b \| PW_i^{new})) \bmod n_0)$ . Then, smart card updates the values of  $N_i$ ,  $A_i$  and  $a_i \oplus A_i$  with  $N_i^{new}$ ,  $A_i^{new}$  and  $a_i \oplus A_i^{new}$ , respectively.

### V. PROTOCOL DESIGN RATIONALES

In this section, we sketch the basic design ideas behind our protocol and show that “two birds” are hit with one stone: the integration of “honeywords” with a “fuzzy-verifier” not only *eliminates the long-standing security-usability tension* but also *achieves security beyond the conventional optimal bound*.

#### A. Basic ideas

To achieve the most essential goal — “truly two-factor security”, a password-protected cryptographically strong long-term secret  $k = \mathcal{H}_0(x \| ID_i \| T_{reg})$  is kept on the smart card, where  $x$  is the server’s secret key. On the one hand, this long-term secret  $k$  can be derived by the server if it knows  $U_i$ ’s identity  $ID_i$  and the time of  $U_i$ ’s registration  $T_{reg}$ . To this end, a table  $\{ID_i, T_{reg}\}$  of registered users is maintained by the server. This, in the mean time, preserves C1. On the other hand,  $k$  is effectively protected by the password so that breaching the smart card security still does not disclose it. However,  $U_i$  herself can reveal it by computing  $k = N_i \oplus \mathcal{H}_0(b \| PW_i^*)$ , where  $N_i$  and  $b$  is stored on the card.

To achieve “local and secure password update” (i.e., C2) and address the smart card loss problem (i.e., C4) at the same time, a verification of the authenticity of the original password before updating the value of  $N_i$  in the smart card is essential. And thus, besides  $N_i$ , some additional parameter(s)

should be stored in the card memory, which may introduce new vulnerabilities, such as offline password guessing and user impersonation. To gain a better insight into the subtleties, now let’s assume an additional parameter  $A_i = \mathcal{H}_0(ID_i \| \mathcal{H}_0(PW_i))$  is stored in the card. Whenever  $U_i$  wants to change her password, firstly she must submit her identity  $ID_i^*$  and password  $PW_i^*$ , then the card checks if  $\mathcal{H}_0(ID_i^* \| \mathcal{H}_0(PW_i^*))$  equals the stored  $A_i$ . One can easily find that  $\mathcal{A}$  can exhaustively search the correct  $(ID_i, PW_i)$  pair in an offline manner once  $A_i$  is extracted, which definitely leads to an offline guessing attack, resulting in the violation of C4. What we have just described directly applies to the schemes in [46], [49], [57], [58], where the parameter  $A_i$  is exactly computed in this insecure manner and thus  $\mathcal{A}$  can obtain the correct  $(ID_i, PW_i)$  pair once  $A_i$  has been revealed under the capability C-2(ii) in Table I.

However, if the parameter  $A_i$  is computed as  $A_i = \mathcal{H}_0(\mathcal{H}_0(ID_i) \oplus \mathcal{H}_0(PW_i)) \bmod n_0$ , one can be assured that there exists  $\frac{|\mathcal{D}_{id}| * |\mathcal{D}|}{n_0} \approx 2^{32}$  candidates of  $(ID, PW)$  pair to frustrate  $\mathcal{A}$  when  $|\mathcal{D}_{id}| = |\mathcal{D}| = 10^6$  [10], [37] and  $n_0 = 2^8$ , where  $|\mathcal{D}_{id}|$  and  $|\mathcal{D}|$  denote the size of the identity space and password space, respectively. Even with the capability of C-01 (i.e., the victim user’s identity has already been learnt),  $\mathcal{A}$  will still be frustrated, because there exist  $\frac{|\mathcal{D}|}{n_0} \approx 2^{12}$  password candidates (as will be empirically established in Sec. V-B). To further exclude the specious passwords from the remaining  $2^{12}$  candidates, there is no other way than launching an online password guessing attack by interacting with  $S$  to determine the exactly correct one. This can be effectively prevented by our introduction of “honeywords” [38] (i.e., a `Honey_List` in this work) into protocol design to *timely* detect the event that the parameters in  $U_i$ ’s card have been extracted. In Sec. V-C, we will demonstrate that this event can be timely detected with an accuracy of  $1 - \frac{1}{2^{80}}$ ; In Sec. VI, we will rigorously show that this is the best strategy that  $\mathcal{A}$  can exploit to obtain the password and thereby to break the protocol. In this manner, we thwart  $\mathcal{A}$  from obtaining the correct  $(ID, PW)$  pair and we call the parameter  $A_i$  calculated through this new method “a fuzzy verifier”. Note that, we do not directly store  $A_i$  on the server side, but instead store a random number  $a_i$  corresponding to  $A_i$  on  $S$  and also store both  $A_i$  and  $a_i \oplus A_i$  on the smart card, in order to eliminate the risks when  $S$  is compromised and  $A_i$  is leaked (i.e., the stolen-verifier attack),

An obvious “side effect” of this “fuzzy verifier” is to achieve timely typo detection (C9). A scheme with C9 ensures that, in case  $U_i$  accidentally keys a wrong  $(ID_i^*, PW_i^*)$  pair, this event can be timely detected, thereby avoiding fruitless time, computation and communication cost and user fatigue.

We also note that an adversary  $\mathcal{A}$  who gets *temporary* access to  $U_i$ ’s smart card may exploit this “security-usability trade-off parameter”  $A_i$ . More specifically,  $\mathcal{A}$  may attempt to change  $U_i$ ’s password to a new one and then: (i) tries to login; or (ii) returns the card back. In the case i, even though  $\mathcal{A}$  somehow guesses  $PW_i$  to be  $PW_i^r$  such that  $\mathcal{H}_0((\mathcal{H}_0(ID_i) \oplus \mathcal{H}_0(b \| PW_i^r)) \bmod n_0)$  equals the stored  $A_i$ , yet there is only a chance of  $\frac{1}{2^{12}}$  that  $PW_i^r$  equals  $PW_i$  (see Sec. V-B). *Only* when  $PW_i = PW_i^r$ ,  $\mathcal{A}$  (and the smart card) can retrieve the correct  $k = \mathcal{H}_0(x \| ID_i \| T_{reg}) = N_i \oplus \mathcal{H}_0(b \| PW_i^r)$  and successfully logins, because a bogus  $k$  will be rejected by the server  $S$  in Step V2 of the protocol and also be detected by our honeywords (see Sec. V-C). The case ii constitutes a denial of service (DoS) attack. If the threshold of consecutive

password change failures is set to five per day (By convention and practice, it is assumed that counter protection of the smart card is in place) and  $\mathcal{H}_0(\dots)$  outputs randomly (which is widely assumed [39]),  $\mathcal{A}$  will succeed with a probability only about  $\frac{1.95}{100} (\approx \frac{5}{n_0} = \frac{5}{2^8})$ . This means  $\mathcal{A}$  will not succeed easily (i.e., with a chance of 50% after 26 days of attack). Even if this DoS attack succeeds,  $U_i$  can restore her card by re-registration.

As discussed above and will be proved in Sec. VI, this DoS attack and the aforementioned online guessing attack are the two greatest threats that  $\mathcal{A}$  poses to our protocol. Fortunately,  $\mathcal{A}$  benefits little from this DoS attack and  $\mathcal{A}$ 's incentive shall be low, while the latter attack can be timely detected and our scheme achieves security beyond the conventional optimal security bound (i.e., beyond  $q_{send}/|\mathcal{D}| + \epsilon$ ).

To avoid password exposure (C3),  $\mathcal{H}_0(b||PW_i)$  instead of  $PW_i$  or  $h(PW_i)$  is submitted to server  $S$ , where  $b$  is a random number unknown to the server  $S$ ; to achieve C6, an entry  $(ID_i, T_{reg})$  corresponding to  $U_i$  is stored in  $S$ 's database, only  $T_{reg}$  needs to be updated when user  $U_i$  revokes her smart card; to avoid clock synchronization (C8), a nonce based mechanism instead of the timestamp based design is preferred to provide the freshness of the messages; to achieve user anonymity (C11), user's real identity  $ID_i$  is concealed in the session-variant pseudo-identity  $CID_i$ , the formal proof for C11 is essentially the same with the Theorem 2 in [56] which is based on the decisional Diffie-Hellman assumption; to achieve forward secrecy (C12), DH key exchange technique is adopted; and C4 and C10 will be further rigorously proved in Sec. VI.

### B. Effectiveness of "fuzzy-verifier"

We now use large-scale real-life passwords to show that our proposed "fuzzy-verifier"  $A_i = \mathcal{H}_0((\mathcal{H}_0(ID_i) \oplus \mathcal{H}_0(b || PW_i)) \bmod n_0)$  is effective in dividing  $\mathcal{A}$ 's password guessing space, leaving adequate candidates for  $\mathcal{A}$  to identify and thus making it possible for "honeywords", as will be shown in Sec. V-C, to bound  $\mathcal{A}$ 's advantage to a low value.

Assume  $\mathcal{A}$  has obtained  $U_i$ 's card and extracted the parameter  $A_i$ . With the knowledge of  $A_i$ ,  $\mathcal{A}$  can reduce the size of her guessing space  $\mathcal{D}$  to  $\frac{|\mathcal{D}|}{n_0}$ . We proceed to show that  $\mathcal{D}/n_0$  is *practically* large enough. It is natural for us to approximate  $\mathcal{D}$  using real-life password accounts *with frequency*; As  $\mathcal{A}$  is clever, when performing an online guessing, she would always try the most likely password candidate first [37], and this attacking strategy is best approximated by the security notion of guessing entropy (GE):

$$G(\mathcal{D}) = \sum_{i=1}^{|\mathcal{D}|} p_i \cdot i$$

where, without loss of generality, each password  $pw_i$  in  $\mathcal{D}$  is assumed to be associated with a probability  $p_i$  and  $p_1 \geq p_2 \geq p_3 \geq \dots$ . It is not difficult to see that  $G(\mathcal{D})$  is the expected number of guesses required to find the correct password  $PW_i$ .

Here we employ four datasets: 32 million Rockyou [14], 30 million Tianya, 16 million Dodonew and 6 million CSDN [16]. The first dataset was hacked in Dec. 2009 from the popular gaming site Rockyou.com, and the later three datasets were hacked in Dec. 2011 from three high-profile web services in China. They all were made publicly available. For illustration, we set  $n_0 = 2^8$ . To be practical, we do not use the entire password datasets to approximate  $\mathcal{A}$ 's guessing space  $\mathcal{D}$ , but

TABLE III. GUESSING ENTROPY (GE) DISTRIBUTIONS OF  $n_0 = 2^8$  PASSWORD POOLS. EACH POOL STEMS FROM THE DIVISION OF PASSWORD SPACE  $\mathcal{D}$  ACCORDING TO OUR FUZZY-VERIFIER  $A_i$ .

Real-life password distributions	Percentage of password pools with $GE \geq 2^{12} = (2^{20}/n_0)$
Rockyou_Top1Million	0.00%
Tianya_Top1Million	10.16%
CSDN_Top1Million	10.54%
Dodonew_Top1Million	14.45%
Rockyou_Top2Million	84.77%
Tianya_Top2Million	96.09%
CSDN_Top2Million	97.66%
Dodonew_Top2Million	98.83%
Rockyou_TopxMillion( $x \geq 3$ )	99.61%
Tianya_TopxMillion( $x \geq 3$ )	100.00%
CSDN_TopxMillion( $x \geq 3$ )	100.00%
Dodonew_TopxMillion( $x \geq 3$ )	100.00%

\*For more details, readers can see <http://wangdingg.weebly.com/fuzzyverifier.html>.

use the most vulnerable distributions (i.e., portions with the top popular passwords). The underlying reason is that: (1)  $\mathcal{A}$  cares about cost-effectiveness and would not try these least popular (low-gain) passwords; (2) If these most vulnerable portions of a dataset can assure satisfactory guessing entropy, these less vulnerable portions would naturally reach the goal.

Table III shows that, when the size of the dataset is no less than 3M, each divided pool indeed can reach a  $GE \geq 2^{12}$  (when  $n_0 = 2^8$ ). This indicates that, even if the parameter  $A_i$  is extracted (or somehow guessed) by  $\mathcal{A}$ , there are still  $2^{12}$  password candidates that  $\mathcal{A}$  has to guess in an online manner, while online guessing can be easily thwarted. This means that our "fuzzy-verifier" is indeed effective for these services with a user-base  $\geq 3M$ , and it will also be effective for services with a smaller scale when we adjust the value of  $n_0$ . Dodonew is the strongest one among the four datasets in term of GE, while Rockyou is the least strong one. This has useful implications: for passwords created under a similar context (e.g., password creation policy) to Dodonew, their guessing space shall be as large as 2M to reach a  $GE \geq 2^{12}$ , while passwords created under a context similar to Rockyou shall be with a space of 3M.

### C. Effectiveness of "fuzzy-verifier" + "honeywords"

In our scheme, to provide the admired property of "local and secure password change" [25],  $U_i$  stores the "fuzzy-verifier"  $A_i = \mathcal{H}_0((\mathcal{H}_0(ID_i) \oplus \mathcal{H}_0(b || PW_i)) \bmod n_0)$  in its card memory. This also facilitates  $\mathcal{A}$  to reduce her password guessing space size from  $|\mathcal{D}|$  to  $\frac{|\mathcal{D}|}{n_0}$  in an offline manner. In the above section, we have shown that our "fuzzy-verifier" is effective in making  $|\mathcal{D}|/n_0$  large enough in practice. One can see that, if  $S$  can effectively detect the event, denoted by Ext, that the parameters in  $U_i$ 's card have been extracted and  $S$  timely suspends the corrupted card,  $\mathcal{A}$  is still prevented from gaining a large advantage in determining the final correct password  $PW_i$  from the  $\frac{|\mathcal{D}|}{n_0}$  candidates by using online guessing. Fortunately, in what follows we show that the integration of "honeywords" (i.e., the items in Honey\_List) with our "fuzzy-verifier" (i.e.,  $A_i$ ) enables our scheme to detect the event Ext with an accuracy of  $1 - \frac{1}{2^{80}}$  after just ten online guessing attempts.

In Step V3 of the Verification phase, whenever the server  $S$  finds a login request that is with the *right*  $A_i$  but with an *erroneous*  $k$ ,  $S$  is with a confidence  $1 - \frac{1}{n_0}$  that the event Ext occurs. Furthermore, if we specify that  $S$  revokes  $U_i$  when such enormous login events occur  $m_0$  times, then  $S$  is with a confidence  $1 - (\frac{1}{n_0})^{m_0}$  (e.g., which equals  $1 - \frac{1}{2^{80}}$  when  $m_0 = 10$  and  $n_0 = 2^8$ ) to be assured that the event Ext

TABLE IV. THE CUMULATIVE PERCENTAGES OF TOP- $x$  MOST POPULAR PASSWORDS OF EACH REAL-LIFE PASSWORD DATASET

Datasets (leaked year)	Language	Total passwords	Unique passwords	Top 1	Top 3	Top 10	Top 10 <sup>2</sup>	Top 10 <sup>3</sup>	Top 10 <sup>4</sup>	Top $\frac{1}{5}$	Top $\frac{1}{10}$	Top $\frac{1}{10^2}$	Top $\frac{1}{10^3}$	Top $\frac{1}{10^4}$
Rockyou (2009)	English	32,581,870	14,326,970	0.89%	1.37%	2.05%	4.55%	11.30%	22.31%	64.82%	57.28%	39.30%	24.24%	12.84%
Phpbbs (2009)	English	255,373	184,341	1.04%	1.80%	2.79%	5.70%	12.89%	27.44%	42.25%	34.05%	15.95%	7.06%	3.42%
Singles.org (2010)	English	16,248	12,233	1.36%	2.10%	3.40%	9.19%	26.35%	86.26%	39.76%	29.09%	10.06%	3.65%	1.36%
Faithwriters (2009)	English	9,708	8,346	0.55%	1.03%	2.17%	7.76%	24.33%	100.00%	31.22%	22.62%	7.06%	1.90%	0.00%
Tianya (2011)	Chinese	30,901,241	12,898,437	3.98%	5.59%	7.43%	11.50%	16.04%	25.78%	66.55%	58.21%	41.19%	27.45%	16.70%
Dodoweb (2011)	Chinese	16,258,891	10,135,260	1.45%	2.15%	3.28%	5.60%	8.59%	13.62%	50.13%	39.97%	22.72%	13.66%	8.61%
Csdn (2011)	Chinese	6,428,277	4,037,605	3.66%	8.15%	10.44%	13.26%	16.54%	23.91%	49.75%	42.66%	28.46%	20.62%	14.97%
Sina weibo (2011)	Chinese	4,730,662	2,828,618	3.74%	4.99%	7.17%	10.24%	13.96%	21.49%	52.17%	43.82%	27.24%	16.51%	11.71%
Gmail (2014)	hybrid	4,926,650	3,132,028	0.97%	1.43%	2.07%	3.88%	8.65%	17.76%	49.14%	41.64%	23.78%	12.63%	5.77%
Mail.ru (2014)	Russian	4,932,688	2,949,616	1.82%	3.06%	4.05%	6.37%	9.94%	17.40%	52.16%	43.88%	24.92%	12.46%	7.81%
Yandex.ru (2014)	Russian	1,261,809	717,202	3.10%	4.98%	7.66%	12.26%	17.43%	26.53%	54.53%	44.29%	24.53%	16.61%	11.55%
Average above	-	9,300,311	4,657,332	<b>2.05%</b>	<b>3.33%</b>	<b>4.78%</b>	<b>8.21%</b>	<b>15.09%</b>	<b>28.25%</b>	50.23%	41.59%	24.11%	14.25%	8.61%
Uniform distribution	-	1000,000	1000,000	0.0001%	0.0003%	0.0010%	0.01%	0.10%	1.00%	20.00%	10.00%	1.00%	0.10%	0.01%

occurs. The choices for the two tradeoff parameters  $m_0$  and  $n_0$  are mainly constrained by the following requirements:

- R1.  $\Pr[\text{Err\_change}] = \frac{1}{n_0}$  shall be as small as possible, where  $\text{Err\_change}$  denotes the event that,  $U_i$  accidentally types an unintended password  $PW_i^*$  when changing password, yet  $\mathcal{H}_0((\mathcal{H}_0(ID_i) \oplus \mathcal{H}_0(b \parallel PW_i^*)) \bmod n_0)$  equals  $A_i$  and  $PW_i$  is unwittingly changed to  $PW_i^*$ .
- R2.  $\Pr[\text{Err\_detect}] = \frac{1}{(n_0)^{m_0}}$  shall be as small as possible, where  $\text{Err\_detect}$  denotes the event that  $S$  incorrectly revokes  $U_i$ 's smart card. That is,  $S$  determines that  $\text{Ext}$  occurs yet actually,  $U_i$ 's card has not been corrupted.
- R3.  $\Pr[\text{Succ\_Ext}] = C' \cdot m_0^{s'}$  shall be as small as possible, where  $\text{Succ\_Ext}$  denotes the event that  $\text{Ext}$  occurs and  $\mathcal{A}$  successfully obtains  $PW_i$  by interacting with  $S$ .

It is critical to observe that: (1) R1 and R2 require  $m_0$  and  $n_0$  to be as *large* as possible, while R3 requires  $m_0$  and  $n_0$  to be as *small* as possible, and thus we need a balance; and (2)  $\Pr[\text{Err\_detect}]$  decreases *exponentially* with  $m_0$ , while  $\Pr[\text{Succ\_Ext}]$  increases *linearly* with  $m_0$ , which means we can timely (and accurately) detect the event  $\text{Ext}$  and, at the meantime, confine  $\mathcal{A}$ 's guessing advantage to the possible minimum by keeping  $m_0$  small enough (e.g.,  $m_0 \in [3, 20]$ ).

In this work, we recommend  $n_0 = 2^8$  and  $m_0 = 10$ . As discussed above, it is acceptable to set  $n_0 = 2^8$ ; Since it is very undesirable to mistakenly suspend a legitimate user's card,  $\Pr[\text{Err\_detect}]$  shall be negligible and when  $m_0 \geq 10$ ,

$$\Pr[\text{Err\_detect}] = \frac{1}{(n_0)^{m_0}} \leq \frac{1}{(2^8)^{10}}$$

This indicates the event  $\text{Ext}$  can be detected with an accuracy of  $1 - \frac{1}{2^{80}}$ . On the other hand, when we set  $m_0 \leq 10$ ,

$$\Pr[\text{Succ\_Ext}] = C' \cdot m_0^{s'} \approx \sum_{j=1}^{m_0} p_j \leq \sum_{j=1}^{10} p_j = 7.43\%$$

where we use the distribution of 31 million Tianya passwords [16] for a concrete example:  $C'=0.062239$  and  $s'=0.155478$ .

We summarize in Table IV the results for 11 real-life password distributions with the guess number (i.e.,  $m_0$ ) varying from 1, 3, 10, 10<sup>2</sup> to 10<sup>4</sup>, as well as varying from 1/10<sup>4</sup> percent to 1/5 percent. Thus, we suggest setting  $m_0=10$ . We emphasize that, the values for  $m_0$  and  $n_0$  can be adjusted to cater for diversified security demands in different systems.

What's most surprising in Table IV is that, with just a handful of online guessing attempts,  $\mathcal{A}$  can obtain a considerable amount of advantages. For instance, the average advantage of  $\mathcal{A}$  will be up to 8.21% by only performing 100 online

guesses (in an optimal order). This is in vast different from the traditional *theoretical* optimal security bound (see [30], [34], [36]):  $q_{\text{send}}/|\mathcal{D}| + \epsilon$ , where  $\mathcal{D}$  is assumed to be uniformly distributed, and  $q_{\text{send}}$  denotes the number of online guessing that  $\mathcal{A}$  engages in. Since  $\mathcal{D}$  is generally assumed to be 10<sup>6</sup> and  $q_{\text{send}}$  to be in thousands [37],  $q_{\text{send}}/|\mathcal{D}|$  will be theoretically over 0.1%. Yet, the actual value is over 15.09%, which is far from a negligible risk level. From the last two rows in Table IV, one can see that there are *two to four orders of magnitude difference* in  $\mathcal{A}$ 's online guessing advantages between the uniform model and the realistic model (which can be well captured by our Zipf model, see Fig. 3). If we had considered *targeted online guessing* [59] and the fact that user passwords become weaker when additional authentication factor are in place [60], this gap will be even larger.

All this highlights that, due to highly skewed password distributions in reality, *the conventional optimal security bound largely underestimates  $\mathcal{A}$ 's advantages and engenders a false sense of security*. It is imperative to prevent  $\mathcal{A}$  from trying moderate number of online guessing to bound  $\mathcal{A}$ 's advantage to a low value. Fortunately, the use of "honeywords" with a "fuzzy-verifier" provides a promising solution to this issue.

In a nutshell, before  $\mathcal{A}$  sets off an alarm of user card corruption with a probability  $\frac{1}{(n_0)^{m_0}}$  (e.g.,  $\frac{1}{2^{80}}$ ) of false positive, she can only mount  $m_0$  ( $m_0 \in [3, 20]$  as suggested) online guessing attempts, which is her best attacking strategy (for further evidence see Sec. VI), while in conventional schemes this figure would generally be in hundreds or even in thousands. In this light, our scheme achieves security (i.e.,  $\Pr[\text{Succ\_Ext}] = C' \cdot m_0^{s'} \approx \sum_{j=1}^{m_0} p_j \leq \sum_{j=1}^{10} p_j$ ) beyond the conventional optimal security bound (i.e.,  $q_{\text{send}}/|\mathcal{D}| + \epsilon$ ), serving as a hedge against human-beings' limited memory.

#### D. Wide applicability of our approach

We proceed to show that our integration of "honeywords" with "fuzzy-verifiers" is a generic one and can be readily applied to existing two-factor schemes for both single-server architecture [26] and multi-server environment [61]. More specifically, a fuzzy-verifier (like our  $A_i$ ) is stored on the user's card memory to provide the usability property "local and secure password change" (i.e., C2), while some "honeywords" (like these items in  $\text{Honey\_List}$ ) are kept on the backend database of the authentication server (or so-called control server in the multi-server environment) to preserve the security goal of "no smart card loss attack" (i.e., C4). Most essentially, "honeywords" enable the system to accurately detect the event that  $\mathcal{A}$  is exploiting the fuzzy-verifier as a oracle to reduce

password space and to thwart  $\mathcal{A}$ 's malicious action in a timely manner (e.g., by rate limiting or locking the account).

In Appendix B, we use two typical schemes, i.e. Tsai et al.'s scheme [32] and Xue et al.'s scheme [58], as case studies to demonstrate exactly how our approach can be integrated into *other two-factor schemes* (for the single-server architecture) and into *two-factor schemes for the multi-server architecture*, respectively. In addition, we also employ Odelu et al.'s scheme [61] to briefly show its applicability to three-factor schemes. As a result, most of the schemes (e.g., [24], [26], [30], [48]) previously subject to the long-standing C2 vs. C4 dilemma now can be relieved by using our proposed approach.

**Summary.** Both theoretical and empirical results show the practicality of integrating “honeywords” [38] with a “fuzzy-verifier” to well balance the useability feature of “local password update” (C2) and the security goal of “no smart loss problem” (C4). This provides thus far the most promising solution to the open problem left by Huang *et al.* [23]: “whether or not there exist secure smart-card-based password authentication protocols and the password-changing phase does not need any interaction with the server”?

## VI. FORMAL SECURITY ANALYSIS

In the following, we show that our scheme is secure in the model defined in Sec. III under the assumptions that the hash function closely behaves like a random oracle and that the computational Diffie-Hellman problem is intractable.

We first provide a formal description of the proposed protocol by specifying the registration phase and the oracles to which the adversary has access. Before the registration phase for security parameter  $\ell$ , algorithm `Init` first runs an algorithm  $\mathcal{G}$  to generate a group  $\mathbb{G}$  of prime order  $q$ , where  $|q| = \ell$ . Next, `Init` generates a generator  $g$  for  $\mathbb{G}$ , four collision-resistant hash functions  $\mathcal{H}_i : \{0, 1\}^* \rightarrow \{0, 1\}^{l_i}$  ( $i = 0, 1, 2, 3$ ), and a long-term private/public key pair  $(x, y = g^x)$  for server  $S$ . Each user  $U_i$  is equipped with a password  $PW_i$  which is drawn from a Zipf-distributed dictionary  $\mathcal{D}$  of size  $|\mathcal{D}|$ . Additionally, when the user  $U_i$  enrolls in the server  $S$ ,  $S$  stores user-specific secret data  $\{N_i, A_i, A_i \oplus a_i\}$  as well as other public parameters into a smart card and issues it to the user  $U_i$ , where  $N_i$  and  $A_i$  are transformations of  $PW_i$  and  $S$ 's private key  $x$ . Further, a formal specification of the `Execute`, `Reveal`, `Corrupt` and `Test` oracles appears in Appendix C.

Before stating the security results, we recall the computational assumption on which the formal security proof relies.

**Computational Diffie-Hellman (CDH) Assumption.** Let  $\mathbb{G}$  be a finite cyclic group of prime order  $q$  generated by an element  $g$ , where the operation is denoted multiplicatively. A  $(t, \epsilon)$ -CDH attacker in  $\mathbb{G}$  is a PPT machine  $\Delta$  running in time  $t$  such that

$$\begin{aligned} \text{Adv}_{g, \mathbb{G}}^{\text{CDH}}(\Delta) &= \Pr[\Delta(g^x, g^y) = g^{xy}] \geq \epsilon \\ \text{Adv}_{g, \mathbb{G}}^{\text{CDH}}(t) &= \max_{\Delta} \{\text{Adv}_{g, \mathbb{G}}^{\text{CDH}}(\Delta)\} \end{aligned}$$

where the probability is taken over the random values  $x$  and  $y$ . The CDH-Assumption states that  $\text{Adv}_{g, \mathbb{G}}^{\text{CDH}}(t) \leq \epsilon$  for any  $t/\epsilon$  not too large.

*Theorem 1:* Let  $\mathbb{G}$  be a representative group,  $\mathcal{D}$  be a password space from which each user password is drawn according to the Zipf's law [8] and  $n_0$  be the “security-usability trade-off parameter”. Let  $\mathcal{P}$  be the proposed authentication scheme

stated in Section IV. Let  $\mathcal{A}$  be a PPT adversary against the semantic security within a time bound  $t$ , with  $q_{\text{send}} (\leq m_0 = 10)$  Send-queries and  $q_{\text{exe}}$  Execution-queries, and making less than  $q_h$  random oracle queries. Then we have

$$\begin{aligned} \text{Adv}_{\mathcal{P}, \mathcal{D}}^{\text{ake}}(\mathcal{A}) &= 2\Pr[\text{Succ}_{\tau}] - 1 + 2(\Pr[\text{Succ}_0] - \Pr[\text{Succ}_{\tau}]) \\ &\leq C' \cdot q_{\text{send}}^{s'} + 12q_h \text{Adv}_{\mathcal{P}}^{\text{CDH}}(t + (q_{\text{send}} + q_{\text{exe}} \\ &\quad + 1) \cdot \tau_e) + \frac{q_h^2 + 8q_{\text{send}}}{2^l} + \frac{(q_{\text{send}} + q_{\text{exe}})^2}{p}, \end{aligned}$$

where we use the Zipf model of the Tianya password distribution in [8], where  $|\mathcal{D}|=12,898,437$ ,  $C'=0.062239$  and  $s'=0.155478$ ;  $n_0 = 2^8$ ;  $\tau_e$  is the computation time for an exponentiation in  $\mathbb{G}$ , and  $l = \min\{l_i\}, i = 0, 1, 2, 3$ .

*Proof.* Let  $\mathcal{A}$  be an adversary against the semantic security of our scheme. Our main idea is to employ  $\mathcal{A}$  to construct probabilistic polynomial-time (PPT) adversaries for each of the underlying primitives (e.g., Hash and CDH intractability) in such a way that if  $\mathcal{A}$  manages to break the semantic security, then at least one of these PPT adversaries succeeds in breaking the security of an underlying primitive. We prove Theorem 1 through a series of hybrid games  $G_n (n = 0, 1, \dots, 8)$ , starting with the real attack  $G_0$  and ending in  $G_8$  where  $\mathcal{A}$ 's advantage is 0, and for which we can bound the difference in  $\mathcal{A}$ 's advantage between any two consecutive games. The detailed proof can be found in Appendix C-A.

*Theorem 2:*  $\mathbb{G}$ ,  $\mathcal{D}$ ,  $n_0$  and  $\mathcal{P}$  are of the same meaning with those of Theorem 1. Let  $\mathcal{A}$  be an adversary against mutual authentication within a time bound  $t$ , with less than  $q_{\text{send}} (\leq m_0 = 10)$  Send-queries and  $q_{\text{exe}}$  Execution-queries, and making less than  $q_h$  random oracle queries. Then we have

$$\begin{aligned} \text{Adv}_{\mathcal{P}, \mathcal{D}}^{\text{auth}}(\mathcal{A}) &\leq C' \cdot q_{\text{send}}^{s'} + \frac{q_h^2 + 8q_{\text{send}}}{2^{l+1}} + \frac{(q_{\text{send}} + q_{\text{exe}})^2}{2p} \\ &\quad + 5q_h \text{Adv}_{\mathcal{P}}^{\text{CDH}}(t + (q_{\text{send}} + q_{\text{exe}} + 1) \cdot \tau_e), \end{aligned}$$

where  $\tau_e$  is the computation time for an exponentiation in  $\mathbb{G}$  and  $l = \min\{l_i\}, i = 0, 1, 2, 3$ .

*Proof.* This proof is similar to that of semantic security. And interested readers are referred to Appendix C-B.

## VII. PERFORMANCE EVALUATION

In this section, we compare the performance and the fulfillment of the criteria among relevant schemes [30], [35], [36], [49], [57], [62]–[64] and our proposed scheme. The comparison results are depicted in Table V. Sixty-seven typical schemes are further evaluated in Appendix A.

Without loss of generality, the security parameter  $n_0$  is assumed to be 32-bit long, the identity  $ID_i$ , password  $PW_i$ , random numbers, timestamp values and output of secure one-way hash function are all recommended to be 128-bit long, while  $y$  and  $g$  are 1024-bit long. Let  $T_H, T_E, T_S$ , and  $T_C$  denote the time complexity for hash, modular exponentiation, symmetric encryption and Chebyshev polynomial, respectively. Other lightweight operations like XOR and  $\parallel$  are omitted.

To have a more intuitive grasp on the computation overhead of our scheme, in Table VI we list the computation time for related cryptographic operations on different platforms. We use a Philips HiPerSmart<sup>TM</sup> card to approximate user device, and



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APPENDIX A  
A COMPARATIVE EVALUATION OF EXISTING TWO-FACTOR AUTHENTICATION SCHEMES

To demonstrate the effectiveness of our framework in practice, we provide a comparative evaluation of 67 two-factor typical schemes by assessing whether the twelve criteria proposed in Section II-B have been met under the security model proposed in Section II-A. Particularly, among these 67 schemes, four were designed before 2004 where the tamper-resistance assumption of the smart cards are generally made. As expected, these early four schemes perform poorly under our new security model (see the bottom of Table A.1), while the recent schemes (e.g., [1]–[6]) generally perform much better. There is a trend that, under our evaluation framework, more recent the scheme is, more desirable the scheme will be.

However, this trend would not be obvious had these 67 schemes been assessed by the existing evaluation frameworks (i.e., [7]–[10]). More specifically, the criteria set in [7], essentially, only includes our criteria C2-C5, and thus one will not see the differences between the schemes proposed in 2010 and the schemes proposed in 2015; The criteria set in [8], [9] concerns “protocol efficiency” and most importantly, no security model is explicitly defined in [8], [9], all this would make these two frameworks virtually impossible to be decidable when assessing a scheme; Using the criteria set in [10] will not reveal the critical “usability-security tension” (discussed later), because it misses the desirable property “freely password change”. All in all, the trend revealed by our framework partially demonstrates the soundness of our evaluation framework.

From the microcosmic point of view, one can see that each criterion is *satisfied* by at least 15 schemes and at the same time, it is *unmet* by at least 7 schemes. *This implies the necessity of each of the twelve criteria.* In addition, there is no scheme that can fulfill all the twelve criteria—the only scheme that can achieve eleven criteria is proposed by Odelu et al. in 2015 [4]. *This suggests the comprehensiveness of our criteria set. This also highlights the needs for more research efforts to design a better scheme.* With a careful examination, one can observe that this scheme suffers from the same “usability-security tension” with other latest schemes (e.g., [2], [3], [5]): the criterion C2 (or C9) and C4 cannot be achieved at the same time. *This suggests the necessity of the separation of C4 from C5, in contrast to the framework in [7].*

It is also worth noting that, in selecting a particular two-factor scheme for inclusion in the comparison Table A.1, we do not necessarily endorse it as better than alternatives that are not included in the table—merely because of that it is reasonably representative, or illuminates in some way what category (from a point view of the development tree where a specific scheme lies, see the history tree Fig. 2 in Section II) it belongs to can achieve. In addition, here we mainly focus on schemes for the single-server architecture because: (1) different architectures/environments may involve quite different attacking vectors and security models, and thus fair comparison is virtually impossible under a single security model; and (2) single-server-architecture-based schemes constitute the basis for schemes that are designed for other more complex architectures/environments (e.g., schemes for wireless sensor networks [11] and mobile networks [12]).

TABLE A.1. A COMPARATIVE EVALUATION OF TWO-FACTOR AUTHENTICATION SCHEMES

Scheme	Year	Ref.	No verifier table (C1)	Password friendly (C2)	No password exposure (C3)	No smart card loss problem (C4)	Resistance to known attacks (C5)	Sound reparability (C6)	Provision of key agreement (C7)	No clock synchronization (C8)	Timely typo detection (C9)	Mutual authentication (C10)	User anonymity (C11)	Forward secrecy (C12)
Lu et al.	2016	[13]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Xie et al.	2016	[14]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Islam et al.	2016	[15]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Muhaya	2015	[11]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Xu-Wu	2015	[2]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Mishra et al.	2015	[3]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Odelu et al.	2015	[4]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Byun	2015	[5]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wang et al.	2015	[6]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Truong et al.	2015	[16]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Djellali et al.	2015	[17]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Mishra et al.	2015	[18]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wu et al.	2015	[19]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Chaudry et al.	2015	[20]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Kumari et al.	2014	[21]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Chen et al.	2014	[22]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Tsai et al.	2013	[23]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Li et al.	2013	[24]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Kumari-Khan	2013	[25]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Sun-Cao	2013	[26]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Lee-Liu	2013	[27]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Islam-Biswas	2013	[28]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Li-Zhang	2013	[29]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Chang et al.	2013	[30]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Li	2013	[31]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Kim-Kim	2012	[32]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Ramasamy-Muniyandi	2012	[33]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wu et al.	2012	[8]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Zhu	2012	[34]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wang-Ma	2012	[35]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Hsieh-Leu	2012	[36]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wen-Li	2012	[37]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wang et al.	2012	[38]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
He et al.	2012	[39]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wang et al.	2011	[40]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Chen et al.	2011	[41]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Khan et al.	2011	[42]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Kim et al.	2011	[43]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Awasthi et al.	2011	[44]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Li-Lee	2011	[45]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Sood et al.	2011	[46]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Li et al.	2010	[47]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Tsai et al.	2010	[48]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Song	2010	[49]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Sood et al.	2010	[50]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Yeh et al.	2010	[51]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Hornig et al.	2010	[52]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Sun et al.	2009	[53]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Hsiang-Shi	2009	[54]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Xu et al.	2009	[55]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Kim-Chung	2009	[56]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Chung et al.	2009	[57]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Ramasamy	2009	[58]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Yang et al.	2008	[7]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Juang et al.	2008	[59]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Chen et al.	2008	[60]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wang et al.	2007	[61]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wang et al.	2007	[62]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Liao et al.	2006	[9]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Lee et al.	2005	[63]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Fan et al.	2005	[64]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Lu-Cao	2005	[65]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Yoon et al.	2004	[66]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Ku et al.	2004	[67]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Wu-Chieu	2003	[68]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Awasthi-Lal	2003	[69]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Sun	2000	[70]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Hwang-Li	2000	[71]	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

\*The present authors have contributed to the following schemes: Wang et al. [6], Wang et al. [38] and Wang-Ma [35]. Interested readers may verify that we have evaluated them impartially.

Finally, during the past five years the present authors have examined more than three hundred two-factor schemes, including over 170 ones for the single server architecture (see

some of our results [72]–[74]), over 100 ones for the multi-server architecture (see some of our results [75]), over 60 ones for mobile networks (see some of our results [76], [77]) and over 50 for the wireless sensor networks (see some of our results [78], [79]). Based on our past cryptanalysis experience, we believe that (1) whenever a specific scheme is identified by us to be *unable* to achieve some criteria, this is sufficiently definite to be true; and (2) when a specific scheme is identified by us to be able to achieve some criteria, this is highly likely to be true, while there may be some probability that we have missed some attacking modes. For instance, we have found that there are at least 8 vastly different attacking modes (e.g., attacking by returning back the extracted card and attacking by exploiting the first/second/third protocol flow) when investigating whether a scheme can resist offline password guessing attack in case the adversary has got access to victim’s card, while this only constitutes part of the total work involved in evaluating just a single criteria C4. *This shows the great difficulty and the mount of manual work entailed when constructing a table like A.1.*

## APPENDIX B WIDE APPLICABILITY OF “FUZZY-VERIFIER” + “HONEYWORDS”

In this Section, we use representative schemes [23], [80], [81] as case studies to demonstrate exactly how our “fuzzy-verifier” and “honeywords” can be integrated into other schemes (and even schemes for other architectures). More specifically, we employ two typical schemes, i.e., Tsai et al.’s scheme [23] and Xue et al.’s scheme [80], to our approach can be integrated into two-factor authentication schemes for the single-server architecture and multi-server environment, respectively. In addition, we also use Odelu et al.’s scheme [81] to briefly show the applicability of our approach to three-factor authentication schemes. After our integration, one can confirm that existing usability-security conflicts in these schemes would be well eliminated. This shed light on how to eliminate the usability-security dilemma in various other schemes (e.g., [7], [8], [53], [82]).

### A. Integration into Tsai et al.’s scheme

Tsai et al.’s scheme [23] has two versions: a two-factor one and a three-factor one. Here we mainly focus on the two-factor version. Their scheme consists of five phases, namely, parameter generation, registration, pre-computation, login and password update. Readers are referred to [23] for details of this scheme, but for ease of comprehension, we will follow its notations as closely as possible.

This scheme exhibits many desirable features over existing schemes, such as user anonymity, high efficiency and formal security proofs, yet it was recently found vulnerable to the smart card loss problem (i.e., unable to achieve C4) [72]. The key point is that, this scheme allows users to change their passwords freely and locally (i.e., able to achieve C2), yet there is no verification of the old password before the update of new password. If an attacker  $\mathcal{A}$  manages to gain temporary access to the smart card of legitimate user  $U_i$  (note that this is a quite realistic assumption in practice), she can easily change the password of user  $U_i$  without any obstacle.

As revealed in [72] and further investigated in Sec. V-A, there are some subtleties and tricks in coping with this problem and before this work, there is no existing solution. Fortunately, our proposed “fuzzy-verifier” and “honeywords” can be integrated into this scheme as follows:

- (1) In the registration phase, besides computing  $V = h(ID_i \| x) \oplus h(PW_i \| b)$ , the server further computes a fuzzy-verifier  $A_i = h((h(ID_i) \oplus h(PW_i \| b)) \bmod n_0)$  for  $U_i$ , and also stores both  $A_i$  and  $A_i \oplus a_i$  in  $U_i$ ’s smart card, where  $h(\cdot)$  is a one-way hash function and  $a_i$  is a random number.  $S$  stores  $\{ID_i, a_i, \text{Honey\_List} = \text{NULL}\}$  in its backend database. The other parts remain the same as in [23].
- (2) In the login phase, after the user  $U_i$  keys her identity  $ID_i$  and password  $PW_i$ , the card computes  $A_i^* = h((h(ID_i) \oplus h(PW_i \| b)) \bmod n_0)$  and verifies the validity of  $A_i^*$  by checking whether  $A_i^*$  equals to the stored  $A_i$ . If the verification holds, it implies the input  $ID_i$  and  $PW_i$  are valid with a probability of  $\frac{n_0-1}{n_0} (\approx \frac{99.61}{100}, \text{ when } n_0=2^8)$ . Otherwise, the smart card rejects. If  $A_i^*$  equals  $A_i$ , the smart card computes  $a_i = (A_i \oplus a_i) \oplus A_i$  and  $C_1 = (ID_i \| a_i \| h(ID_i \| x) \| (h(ID_i \| x) \oplus N_{C2})) \oplus h_1(c)$ . The other parts remain the same as in [23].
- (3) In Step 3 of the login phase, after the server  $S$  receives the login request  $\{C_1, e\}$  from  $U_i$ ,  $S$  derives  $ID_i' \| a_i' \| h(ID_i \| x)'$  from  $C_1$ , and checks whether  $a_i'$  equals the stored  $a_i$ .  $S$  rejects if they are not equal. Otherwise,  $S$  checks whether the derived  $h(ID_i \| x)'$  equals the computed  $h(ID_i \| x)$ . If they are equal,  $S$  proceeds to the next step. If they are unequal,  $S$  now knows that  $a_i' = a_i$  but  $h(ID_i \| x)' \neq h(ID_i \| x)$ , implying that there is a  $1/2^{n_0}$  probability that  $U_i$ ’s card has been corrupted. Accordingly,  $S$  performs either (1) inserts  $h(ID_i \| x)'$  into  $\text{Honey\_List}$  when there are less than  $m_0$  (e.g.,  $m_0 = 10$ ) items in  $\text{Honey\_List}$ ; or (2) suspends  $U_i$ ’s card (i.e., when there are  $m_0$  items in  $\text{Honey\_List}$ ) until  $U_i$  re-registers. The other parts remain the same as in [23].
- (4) In the password-change phase, the user  $U_i$  keys *twice* her identity  $ID_i$  and password  $PW_i$ . If  $U_i$  accidentally keys two unmatched  $(ID_i, PW_i)$  pairs, which is quite realistic in mobile device use cases, the card rejects. If they are match, the card computes  $A_i^* = h((h(ID_i) \oplus h(PW_i \| b)) \bmod n_0)$  and verifies the validity of  $A_i^*$  by checking whether  $A_i^*$  equals to the stored  $A_i$ . If the verification holds, it implies the input  $ID_i$  and  $PW_i$  are valid with a probability of  $\frac{n_0-1}{n_0} (\approx \frac{99.61}{100}, \text{ when } n_0=2^8)$ . Otherwise, the smart card rejects. Then, the card asks the user to submit a new password  $PW_i^{new}$  and computes  $V^{new} = V \oplus h(PW_i \| b) \oplus h(PW_i^{new} \| b)$ ,  $A_i^{new} = h((h(ID_i) \oplus h(PW_i^{new} \| b)) \bmod n_0)$  and  $a_i^{new} = (A_i \oplus a_i) \oplus A_i \oplus A_i^{new}$ . Then, smart card updates the values of  $V$ ,  $A_i$  and  $a_i$  with  $V^{new}$ ,  $A_i^{new}$  and  $a_i^{new}$ , respectively.

Our above amendments are essentially based on the idea illustrated in Sec. V of the main text: we first force the attacker  $\mathcal{A}$  to *has to* launch an online password guessing attack by interacting with  $S$  in order to determine the exactly correct password, and then design ways to enable  $S$  to *timely* detect the event that the parameters in  $U_i$ ’s card have been extracted and used by  $\mathcal{A}$  to perform an online guessing attack. While there may be other ways to conquer this usability-security tension,

we for the first time show the potential and provide a concrete solution to conquer this usability-security tension in [23].

### B. Integration into Xue et al.'s scheme

Xue et al.'s scheme [80] is designed for the multi-server architecture, which is suitable for environments where users need to access more than one service server but only maintain one password and one smart card. Their scheme involves three participants: user  $U_i$ , service server  $S_j$  and control server  $CS$ . It consists of three phases (i.e., registration, login and authentication) and two phases (i.e., password update and dynamic identity update). Readers are referred to [80] for details of this scheme, but for ease of comprehension, we will follow its notations as closely as possible.

In  $U_i$ 's card memory, there are two parameters stored: a random number  $b$  and  $C_i = h(ID_i \| A_i) = h(ID_i \| h(b \| PW_i))$ . These two parameters together can be used to support the property "timely typo detection" (i.e., C9). One can see that, if an adversary  $\mathcal{A}$  obtains the card and extracts these two parameters,  $\mathcal{A}$  can determine  $U_i$ 's password  $PW_i$  as follows:

- Step 1. Guesses the value of  $ID_i$  to be  $ID_i^*$  from dictionary space  $\mathcal{D}_{id}$  and the value of  $PW_i$  to be  $PW_i^*$  from dictionary space  $\mathcal{D}_{pw}$ ;
- Step 2. Computes  $C_i^* = h(ID_i^* \| A_i) = h(ID_i^* \| h(b \| PW_i^*))$ , where  $b$  is revealed from  $U_i$ 's card;
- Step 3. Checks the correctness of  $(ID_i^*, PW_i^*)$  by comparing if the computed  $C_i^*$  equals the extracted  $C_i$ ;
- Step 4. Repeats Step 1, 2 and 3 of this procedure until the correct value of  $(ID_i^*, PW_i^*)$  is found.

The time complexity of the above attacking procedure is  $\mathcal{O}(|\mathcal{D}_{id}| * |\mathcal{D}_{pw}| * 2T_H)$ , where  $T_H$  is the running time for Hash operation. In reality, the dictionary size is very restricted, e.g.,  $|\mathcal{D}_{id}| \leq |\mathcal{D}_{pw}| \leq 10^6$  [83], [84]. Further, according to the timings in Table B.1,  $\mathcal{A}$  may determine the password in about 17.63 days on a common PC.

TABLE B.1. COMPUTATION EVALUATION OF RELATED OPERATIONS ON COMMON PCs

Experimental platform (common PCs)	Modular Exp. $T_E$ ( $ n  = 512$ )	Hash operation $T_H$ (SHA-1)	Other lightweight oper.(e.g., XOR,   )
Intel T5870 2.00 GHz	2.573 ms	2.437 $\mu$ s	0.011 $\mu$ s
Intel i5750 2.66 GHz	2.106 ms	1.980 $\mu$ s	0.009 $\mu$ s
Pentium IV 3.06 GHz	1.676 ms	1.523 $\mu$ s	0.008 $\mu$ s

In addition, *user ID generally cannot be considered as a secret and actually, it is often publicly available*. Thus, there is a high probability for  $\mathcal{A}$  to learn the user's identity  $ID_i$  other than guessing it. In this light, the above attack will be more practical. What's more, high performance computers are quite common those days and cheap cloud computing services are also easily available (e.g., Amazon EC2 [85]). All this indicates that the above attack is effective even if  $\mathcal{A}$  has to figure out both  $ID_i$  and  $PW_i$  simultaneously.

To conquer this vulnerability while still preserving C9, the *definite* password verifier  $C_i = h(ID_i \| h(b \| PW_i))$  shall be changed to a *fuzzy* verifier  $FC_i = h(ID_i \| h(b \| PW_i) \bmod n_0)$ . Furthermore, some "honeywords" shall be kept by  $CS$  to detect the user card breach event. More specifically, our proposed "fuzzy-verifier" and "honeywords" can be integrated into this scheme as follows:

- (1) In the registration phase,  $U_i$  does not compute  $C_i$  but instead computes a fuzzy-verifier  $FC_i = h(h(ID_i \| h(b \| PW_i)) \bmod n_0)$ , and stores  $FC_i$  and  $FC_i \oplus a_i$  in  $U_i$ 's smart card, where  $h(\cdot)$  is a one-way hash function and  $a_i$  is a random number. The control server  $CS$  stores  $\{ID_i, a_i, \text{Honey\_List}=\text{NULL}\}$  in its backend database. The other parts remain the same as in [23].
- (2) In the login phase, after the user  $U_i$  keys her identity  $ID_i$  and password  $PW_i$ , the card computes  $FC_i = h(h(ID_i^* \| h(b \| PW_i^*)) \bmod n_0)$  and verifies the validity of  $FC_i^*$  by checking whether  $FC_i^*$  equals to the stored  $FC_i$ . If the verification holds, it implies the input  $ID_i$  and  $PW_i$  are valid with a probability of  $\frac{n_0-1}{n_0}$  ( $\approx \frac{99.61}{100}$ , when  $n_0=2^8$ ). Otherwise, the smart card rejects. If  $FC_i^*$  equals  $FC_i$ , the smart card computes  $a_i = (FC_i \oplus a_i) \oplus FC_i$  and  $CID_i = (ID_i \| a_i \| B_i) \oplus h(B_i \| N_{i1} \| TS_i \| "00")$ . The other parts remain the same as in [80].
- (3) In Step 3 of the login phase, after the control server  $CS$  receives the login request  $\{F_i, P_{ij}, CID_i, G_i, PID_i, TS_i, J_i, K_i, L_i, M_i, PSID_j\}$  from  $S_j$ ,  $CS$  derives  $ID_i^* \| a_i^* \| B_i^*$  from  $CID_i$ , and checks whether  $a_i^*$  equals the stored  $FC_i$ .  $S$  rejects if they are not equal. Otherwise,  $CS$  checks whether the derived  $B_i^*$  equals the computed  $h(PID_i \| x)$ . If they are equal,  $CS$  proceeds to the next step. If they are unequal,  $CS$  now knows that  $a_i^* = a_i$  but  $h(PID_i \| x)' \neq h(PID_i \| x)$ , implying that there is a  $1/2^{n_0}$  probability that  $U_i$ 's card has been corrupted. Accordingly,  $CS$  performs either (1) inserts  $h(PID_i \| x)'$  into Honey\_List when there are less than  $m_0$  (e.g.,  $m_0 = 10$ ) items in Honey\_List; or (2) suspends  $U_i$ 's card (i.e., when there are  $m_0$  items in Honey\_List) until  $U_i$  re-registers. The other parts remain the same as in [80].
- (4) In the password-change phase, the user  $U_i$  keys *twice* her identity  $ID_i$  and password  $PW_i$ . If  $U_i$  accidentally keys two unmatched  $(ID_i, PW_i)$  pairs, which is quite realistic in mobile device use cases, the card rejects. If they are match, the card computes  $FC_i^* = h(h(ID_i^* \| h(b \| PW_i^*)) \bmod n_0)$  and verifies the validity of  $FC_i^*$  by checking whether  $FC_i^*$  equals to the stored  $FC_i$ . If the verification holds, it implies the input  $ID_i$  and  $PW_i$  are valid with a probability of  $\frac{n_0-1}{n_0}$  ( $\approx \frac{99.61}{100}$ , when  $n_0=2^8$ ). Otherwise, the smart card rejects. Then, the card asks the user to submit a new password  $PW_i^{new}$  and computes  $FC_i^{new} = h(h(ID_i \| h(b \| PW_i^{new})) \bmod n_0)$ ,  $D_i^{new} = D_i \oplus h(h(ID_i \| b) \oplus h(b \| PW_i) \oplus h(b \| PW_i^{new}))$  and  $a_i^{new} = (A_i \oplus a_i) \oplus A_i \oplus A_i^{new}$ . Then, smart card updates the values of  $FC_i$ ,  $D_i$  and  $a_i$  with  $FC_i^{new}$ ,  $D_i^{new}$  and  $a_i^{new}$ , respectively.

### C. Integration into Odelu et al.'s scheme

Odelu et al.'s scheme [81] is three-factor scheme designed for the multi-server architecture. As we have said earlier, this kind of scheme is suitable for environments where users need to access more than one service server but only maintain one password and one smart card (as well as her fingerprint). Their scheme involves three participants: user  $U_i$ , service server  $S_j$  and registration center  $RC$  (which serves as the same role of the control server  $CS$  in Xue et al.'s scheme [80]). It consists of four phases (i.e., initialization, registration, login and authentication) and two phases (i.e., password update, and Revocation and re-registration). Readers are referred to [81]

for details of this scheme, but for ease of comprehension, we will follow its notations as closely as possible.

In  $U_i$ 's card memory, there are three parameters stored: an auxiliary string  $\theta_i$ ,  $s_i = H(k_i \| ID_i \| H(pw_i \| \sigma_i))$  and  $z_i = k_i \oplus H(pw_i \| \sigma_i)$ . It is not difficult to see that, Odelu et al.'s scheme [81] cannot provide truly "three-factor security", which is the most essential goal of a three-factor scheme. More specifically,  $U_i$ 's password factor can be offline guessed by using  $s_i$  as a comparison target if the other two authentication factors (i.e., smart card and biometric) have been breached. Odelu et al. have realized this issue and pointed out that their scheme "can achieve the three-factor authentication by removing the hash value  $s_i$  from the smart-card" and "In that case, the password change will not be possible locally."

Fortunately, our "fuzzy-verifier" and "honeywords" can be integrated into this scheme to ensure that three-factor security can still be achieved while preserving the property of "local and secure password change". The details are as follows:

- (1) In the registration phase, the registration center  $RC$  computes  $s_i$  as  $s_i = H((k_i \| ID_i \| H(pw_i \| \sigma_i)) \bmod n_0)$ , and stores  $s_i$  and  $s_i \oplus a_i$  in  $U_i$ 's smart card, where  $H(\cdot)$  is a one-way hash functions.  $RC$  stores  $\{ID_i, a_i, H(ID_i \| k), r_i, \text{Honey\_List}=\text{NULL}\}$  in its backend database. The other parts remain the same as in [81].
- (2) In the login phase, after the user  $U_i$  keys her identity  $ID_i$  and password  $PW_i$  and imprints her personal biometrics  $B'_i$  at the sensor. Then, smart card computes  $\sigma'_i = \text{Rep}(B'_i, \theta_i)$  and  $k'_i = z'_i \oplus H(pw'_i \| \sigma'_i)$  and checks whether  $s'_i = H((k'_i \| ID_i \| H(pw'_i \| \sigma'_i)) \bmod n_0)$  equals the stored  $s_i$ . If the verification holds, it implies the input  $PW_i$  is valid with a probability of  $\frac{n_0-1}{n_0} (\approx \frac{99.61}{100}, \text{ when } n_0=2^8)$ . Otherwise, the smart card rejects. If  $s'_i$  equals  $s_i$ , the smart card computes  $C_1 = E_{K_{1x}}[ID_i \| a_i \| k_i \| SID_j \| s_j \| n_1]$ . The other parts remain the same as in [81].
- (3) In Step AK2 of the Authentication phase, after the the registration center  $RC$  receives the login request  $\{C_1, X, h_1, C_2, h_2\}$  from  $S_j$ ,  $RC$  derives  $ID'_i \| a'_i \| k'_i$  from  $C_1$ , and checks whether  $a'_i$  equals the stored  $a_i$ .  $RC$  rejects if they are not equal. Otherwise,  $RC$  checks whether the derived  $k'_i$  equals the computed  $k_i = H(ID_i \| k \| r_i \| H(ID_i \| k))$ . If they are equal,  $RC$  proceeds to the next step. If they are unequal,  $RC$  now knows that  $a'_i = a_i$  but  $k'_i \neq H(ID_i \| k \| r_i \| H(ID_i \| k))$ , implying that there is a  $1/2^{n_0}$  probability that  $U_i$ 's card has been corrupted. Accordingly,  $RC$  performs either (1) inserts  $k'_i$  into  $\text{Honey\_List}$  when there are less than  $m_0$  (e.g.,  $m_0 = 10$ ) items in  $\text{Honey\_List}$ ; or (2) suspends  $U_i$ 's card (i.e., when there are  $m_0$  items in  $\text{Honey\_List}$ ) until  $U_i$  re-registers. The other parts remain the same as in [81].
- (4) In the password-change phase, the user  $U_i$  keys *twice* her identity  $ID_i$  and password  $PW_i$ . If  $U_i$  accidentally keys two unmatched  $(ID_i, PW_i)$  pairs, which is quite realistic in mobile device use cases, the card rejects. If they are match,  $U_i$  inputs her biometric  $B'_i$  and the card computes  $\sigma'_i = \text{Rep}(B'_i, \theta_i)$ ,  $s'_i = H((k_i \| ID_i \| H(pw_i \| \sigma'_i)) \bmod n_0)$  and verifies the validity of  $s'_i$  by checking whether  $s'_i$  equals to the stored  $s_i$ . If the verification holds, it implies the input  $PW_i$  is valid with a probability of  $\frac{n_0-1}{n_0} (\approx \frac{99.61}{100}, \text{ when } n_0=2^8)$ . Otherwise, the smart card rejects. Then, the

card asks the user to submit a new password  $PW_i^{new}$  and computes  $s_i^{new} = H((k_i \| ID_i \| H(pw_i^{new} \| \sigma_i)) \bmod n_0)$ ,  $z_i^{new} = z_i \oplus H(PW_i \| \sigma_i) \oplus H(PW_i^{new} \| \sigma_i)$  and  $a_i^{new} = (s_i \oplus a_i) \oplus s_i \oplus s_i^{new}$ . Then, smart card updates the values of  $s_i$ ,  $z_i$  and  $a_i$  with  $s_i^{new}$ ,  $z_i^{new}$  and  $a_i^{new}$ , respectively.

## APPENDIX C

### FORMAL SECURITY ANALYSIS OF OUR SCHEME

#### A. Proof of Theorem 1

**Proof.** In the proof below, we do not consider forward-secrecy for simplicity. We incrementally define a sequence of games starting at the real attack game  $G_0$  and ending up with  $G_8$ . For each game  $G_n$  ( $n=0,1, \dots, 8$ ), we define the following events:

- **Succ<sub>n</sub>** occurs if  $\mathcal{A}$  correctly guesses the bit  $c$  involved in the Test-query.
- **AskPara<sub>n</sub>** occurs if  $\mathcal{A}$  correctly computes the parameter  $k$  by asking a hash query  $\mathcal{H}_0$  on  $b \| PW_i$  or  $x \| ID_i \| T_{reg}$ .
- **AskAuth<sub>n</sub>** occurs if  $\mathcal{A}$  correctly computes the parameter  $k$  and asks a hash query  $\mathcal{H}_1$  (or  $\mathcal{H}_2$ ) on  $ID_i \| ID_S \| Y_1 \| C_2 \| k \| K$ , where  $K$  is  $K_U$  or  $K_S$ .
- **AskH<sub>n</sub>** occurs if the adversary asks a hash query  $\mathcal{H}_i$  ( $i = 1, 2, 3$ ) on  $ID_i \| ID_S \| Y_1 \| C_2 \| k \| K$ , where  $K$  is  $K_U$  or  $K_S$ .

**Game  $G_0$ :** This game corresponds to the real attack, in the random oracle model. By definition, we have

$$\text{Adv}_{\mathcal{P}}^{\text{ake}}(\mathcal{A}) = 2\text{Pr}[\text{Succ}_0] - 1. \quad (1)$$

**Game  $G_1$ :** In this game, we simulate the hash oracles  $\mathcal{H}_i$  ( $i=0,1,2$  and 3, but also four additional hash functions  $\mathcal{H}'_i$  that will appear in Game  $G_7$ ) as usual by maintaining a hash list  $\Lambda_{\mathcal{H}}$  (and another list  $\Lambda_{\mathcal{A}}$  containing the hash-queries asked by the adversary itself). We also simulate all the instances, as the real players would do, for the Send-queries and for the Execute, Reveal, Corrupt and Test-queries (see Figure C.1).

From this simulation, one can easily see that this game is perfectly indistinguishable from the real attack. Hence,

$$|\text{Pr}[\text{Succ}_1] - \text{Pr}[\text{Succ}_0]| = 0 \quad (2)$$

**Game  $G_2$ :** For an easier analysis, in this game, we simulate all oracles as in game  $G_1$  except that we cancel games in which some (unlikely) collisions appear:

- collisions on the partial transcripts  $((C_1, M_i, CAK_i, CID_i), (C_2, C_3), C_4)$ . Note that transcripts involve at least one honest party, and thus one of  $C_1$  or  $C_2$  is truly uniformly distributed;
- collisions on the output of hash queries.

Both probabilities are bounded by the birthday paradox:

$$|\text{Pr}[\text{Succ}_2] - \text{Pr}[\text{Succ}_1]| \leq \frac{(q_{send} + q_{exe})^2}{2p} + \frac{q_h^2}{2^{l+1}} \quad (3)$$

where  $l = \min\{l_i\}, i = 0, 1, 2, 3$ .

**Game  $G_3$ :** We define game  $G_3$  by aborting the game where in the adversary may have lucky in guessing the correct authenticator  $C_3$  or  $C_4$  (that is, without asking the corresponding hash query  $\mathcal{H}_1$  or  $\mathcal{H}_2$ ). Since  $C_1$  and  $C_2$  did not appear in a previous session (since the Game  $G_2$ ), this happens only if

<p>For a hash-query <math>\mathcal{H}_i(q)</math> or <math>\mathcal{H}'_i(q)</math> (with <math>i \in \{0, 1, 2, 3\}</math>), such that a record <math>(i, q, r)</math> appears in <math>\Lambda_{\mathcal{H}}</math>, the answer is <math>r</math>. Otherwise the answer <math>r</math> is defined according to the following rule:</p> <p>► <b>Rule <math>\mathcal{H}^{(i)}</math></b> — Choose a random element <math>r \in \{0, 1\}^l</math>.</p> <p>The record <math>(i, q, r)</math> is added to <math>\Lambda_{\mathcal{H}}</math>. If the query is directly asked by the adversary, one adds <math>(i, q, r)</math> to <math>\Lambda</math>.</p>
<p>We answer to the Send-queries to the client as follows:</p> <p>— A Send <math>(U', \text{Start})</math>-query is processed according to the following rule:</p> <p>► <b>Rule <math>\mathbf{U1}^{(1)}</math></b> — Choose <math>\theta \in_{\mathcal{R}} \mathbb{Z}_p</math> and compute <math>C_1 = g^\theta</math>, <math>Y_1 = y^\theta</math>, <math>k = N_i \oplus \mathcal{H}_0(b \  PW_i)</math>, <math>M_i = \mathcal{H}_0(Y_1 \  k \  CAK_i \  CID_i)</math>, <math>CAK_i = (a_i \  k) \oplus \mathcal{H}_0(Y_1 \  C_1)</math>, and <math>CID_i = ID_i \oplus \mathcal{H}_0(C_1 \  Y_1)</math>. Then the query is answered with <math>(C_1, M_i, CID_i, CAK_i)</math>, and the client instance goes to an expected state.</p> <p>— If the client instance <math>U^i</math> is in an expected state, a query Send <math>(U^i, (C_2, C_3))</math> is processed by computing the session key and producing an authenticator. We apply the following rules:</p> <p>► <b>Rule <math>\mathbf{U2}^{(1)}</math></b> — Compute <math>K_U = C_2^g</math> and <math>C_3^* = \mathcal{H}_0(ID_i \  ID_S \  Y_1 \  C_2 \  k \  K_U)</math>. Reject if the computed <math>C_3^*</math> is not equal to the received <math>C_3</math>. Otherwise moves on.</p> <p>► <b>Rule <math>\mathbf{U3}^{(1)}</math></b> — Compute the authenticator <math>C_4 = \mathcal{H}_2(ID_i \  ID_S \  Y_1 \  C_2 \  k \  K_U)</math> and the session key <math>sk_U = \mathcal{H}_3(ID_i \  ID_S \  Y_1 \  C_2 \  k \  K_U)</math>.</p> <p>Finally the query is answered with <math>C_4</math>, the client instance accepts and terminates. Our simulation also adds <math>((C_1, M_i, CID_i, CAK_i), (C_2, C_3), C_4)</math> to <math>\Lambda_{\Psi}</math>. The variable <math>\Lambda_{\Psi}</math> keeps track of the exchanged messages.</p>
<p>We answer to the Send-queries to the server as follows:</p> <p>— A Send <math>(S', (C_1, M_i, CID_i, CAK_i))</math>-query is processed according to the following rule:</p> <p>► <b>Rule <math>\mathbf{S1}^{(1)}</math></b> — Compute <math>Y = C_1^x</math> and <math>ID_i = CID_i \oplus \mathcal{H}_0(C_1 \  Y_1)</math>, and rejects <math>ID_i</math> is not valid. Reject if the computed <math>M_i^* = \mathcal{H}_0(Y_1 \  k \  CID_i \  CAK_i)</math> is not equal to the received <math>M_i</math>. Compute <math>k = \mathcal{H}_0(x \  ID_i \  T_{reg})</math> and <math>a_i' \  k' = CAK_i \oplus \mathcal{H}_0(Y_1 \  C_1)</math>. Reject if <math>a_i' \neq \text{stored } a_i</math>. Suspend <math>U_j</math> if <math> Honey\_list  \geq m_0 \wedge (k' \neq k) \wedge (a_i' \neq a_i)</math>. Insert <math>k</math> into <math>Honey\_list</math> if <math> Honey\_list  &lt; m_0</math>.</p> <p>► <b>Rule <math>\mathbf{S2}^{(1)}</math></b> — Choose a random exponent <math>\varphi \in_{\mathcal{R}} \mathbb{Z}_p</math>. Compute <math>K_S = C_1^\varphi</math>, <math>C_2 = g^\varphi</math>, and <math>C_3 = \mathcal{H}_0(ID_i \  ID_S \  Y_1 \  C_2 \  k \  K_S)</math>. Finally, the query is answered with <math>(C_2, C_3)</math> and the server instance goes to an expected state.</p> <p>— If the server instance <math>S^i</math> is in an expected state, a query Send <math>(S^i, C_4)</math> is processed according to the following rules:</p> <p>► <b>Rule <math>\mathbf{S3}^{(1)}</math></b> — Compute <math>C_4^* = \mathcal{H}_2(ID_i \  ID_S \  Y_1 \  C_2 \  k \  K_S)</math>, and check whether <math>C_4^* = C_4</math>. If the equality does not hold, the server instance terminates without acceptance. If equality holds, the server instance accepts and goes on, applying the following rule:</p> <p>► <b>Rule <math>\mathbf{S4}^{(1)}</math></b> — Compute the session key <math>sk_S = \mathcal{H}_3(ID_i \  ID_S \  Y_1 \  C_2 \  k \  K_S)</math>. Finally, the server instance terminates.</p>
<p>An Execute <math>(U', S')</math>-query is processed using a successive simulations of the Send-queries: <math>(U', (C_1, M_i, CID_i)) \leftarrow \text{Send}(U', \text{Start})</math>, <math>(C_2, C_3) \leftarrow \text{Send}(S', (C_1, M_i, CID_i, CAK_i))</math> and <math>C_4 \leftarrow \text{Send}(U^i, (C_2, C_3))</math>, and outputting the transcript <math>((C_1, M_i, CID_i, CAK_i), (C_2, C_3), C_4)</math>.</p>
<p>A Corrupt <math>(U, a)</math>-query returns the password <math>PW_i</math> if <math>a=1</math>; returns <math>\{N_i, A_i, b\}</math> if <math>a=2</math>. (Since we do not consider forward secrecy in this proof, no Corrupt <math>(S, 1)</math>-query occurs.)</p>
<p>A Reveal <math>(I)</math>-query returns the session key <math>(sk_U</math> or <math>sk_S)</math> computed by the instance <math>I</math> (if the latter has accepted).</p>
<p>A Test <math>(I)</math>-query first gets <math>sk</math> from Reveal <math>(I)</math>, and flips a coin <math>c</math>. If <math>c=1</math>, we return the value of the session key <math>sk</math>, otherwise we return a random value drawn from <math>\{0, 1\}^l</math>.</p>

Fig. C.1. Simulation of the queries in our scheme

the authenticator  $C_3$  (or  $C_4$ ) had been correctly guessed by  $\mathcal{A}$  without asking  $\mathcal{H}_1$  (or  $\mathcal{H}_2$ ):

$$|\Pr[\text{Succ}_3] - \Pr[\text{Succ}_2]| \leq \frac{q_{\text{send}}}{2^l} \quad (4)$$

**Game  $\mathbf{G}_4$ :** We define game  $\mathbf{G}_4$  by aborting the game where in the adversary may have lucky in guessing the correct parameter  $k$  (i.e., without asking the corresponding query). We reach this aim by modifying the way the participants process the queries. We use the rule as follows:

- **Rule  $\mathbf{U3}^{(4)}$**  — Look for a record  $(0, * \| ID_i \| *, k)$  in  $\Lambda_{\mathcal{A}}$ . If such a record does not exist, we abort the game. Otherwise, compute  $C_4 = \mathcal{H}_2(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_U)$  and the session key  $sk_U = \mathcal{H}_3(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_U)$ .
- **Rule  $\mathbf{S3}^{(4)}$**  — computes  $C_4^* = \mathcal{H}_2(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_S)$  and then checks if  $C_4^*$  equals the received value of  $C_4$ . If this verification holds, the server looks for a record  $(0, *, P_i)$  in  $\Lambda_{\mathcal{A}}$  and a record  $((C_1, M_i, CAK_i, CID_i), (C_2, C_3), C_4)$  in  $\Lambda_{\Psi}$ . If such records do not exist, we abort the game.

Since  $C_1$  and  $C_2$  did not appear in a previous session (since the Game  $\mathbf{G}_2$ ), this happens only if the parameter  $k$  had been correctly guessed by the adversary without asking  $\mathcal{H}_0$ :

$$|\Pr[\text{Succ}_4] - \Pr[\text{Succ}_3]| \leq \frac{q_{\text{send}}}{2^{l_0}} \quad (5)$$

**Game  $\mathbf{G}_5$ :** We define this game by aborting the game where in the adversary may have computed the correct parameter  $k$

and impersonate as a client or server. We reach this aim by modifying the way the participants process the queries. We use the rule as follows:

- **Rule  $\mathbf{U2}^{(5)}$**  — Look for a record  $(0, * \| ID_i \| *, k)$  in  $\Lambda_{\mathcal{A}}$ . If such a record exists, we abort the game. Otherwise, compute  $K_U = (C_2)^u \bmod p$ ,  $C_3^* = \mathcal{H}_1(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_U)$ , and compare  $C_3^*$  with the received  $C_3$ . If the equality does not hold, terminate without acceptance. Otherwise, move on.
- **Rule  $\mathbf{S3}^{(5)}$**  — computes  $C_4^* = \mathcal{H}_2(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_S)$  and then checks if  $C_4^*$  equals the received value of  $C_4$ . If this verification holds, the server looks for a record  $(0, *, P_i)$  in  $\Lambda_{\mathcal{A}}$  and a record  $((C_1, M_i, CAK_i, CID_i), (C_2, C_3), C_4)$  in  $\Lambda_{\Psi}$ . If such records exist, we abort the game.

The two games  $\mathbf{G}_5$  and  $\mathbf{G}_4$  are perfectly indistinguishable unless the event  $\text{AskPara}_5$  occurs:

$$|\Pr[\text{Succ}_5] - \Pr[\text{Succ}_4]| \leq \Pr[\text{AskPara}_5] \quad (6)$$

To upper bound  $\Pr[\text{AskPara}_5]$ , the parameter  $k$  is assumed to be correctly computed by  $\mathcal{A}$  in all the ensuing games.

**Game  $\mathbf{G}_6$ :** We define this game by aborting the game where in the adversary may have computed the correct authenticator  $C_3$  or  $C_4$  (that is, by asking the corresponding hash query  $\mathcal{H}_1$  or  $\mathcal{H}_2$ ) and impersonate as a client or server. We reach this aim by modifying the way the participants process the queries. We use the rule as follows:

- **Rule  $\mathbf{U3}^{(6)}$**  — Check if  $(1, ID_i \| ID_S \| Y_1 \| C_2 \| k \| *, C_3) \in \Lambda_{\mathcal{A}}$ . If it holds, we abort the game. Otherwise, the user goes on to compute  $C_4$  and  $sk_U$ .
- **Rule  $\mathbf{S3}^{(6)}$**  — computes  $C_4^* = \mathcal{H}_2(ID_i \| ID_S \| Y_1 \| C_2 \| k \| K_S)$  and then checks if  $C_4^*$  equals the received value of  $C_4$ . If this verification holds, the server looks for a record  $(0, *, P_i)$  or  $(2, ID_i \| ID_S \| Y_1 \| C_2 \| k \| *, C_4)$  in  $\Lambda_{\mathcal{A}}$ , and a record  $((C_1, CAK_i, CID_i), (C_2, C_3), C_4)$  in  $\Lambda_{\Psi}$ . If such records exist, we abort the game.

The two games  $\mathbf{G}_6$  and  $\mathbf{G}_5$  are perfectly indistinguishable unless event  $\text{AskAuth}_6$  occurs:

$$|\Pr[\text{Succ}_6] - \Pr[\text{Succ}_5]| \leq \Pr[\text{AskAuth}_6] \quad (7)$$

$$|\Pr[\text{AskPara}_6] - \Pr[\text{AskPara}_5]| \leq \Pr[\text{AskAuth}_6] \quad (8)$$

**Game  $\mathbf{G}_7$ :** In this game, we replace the random oracles  $\mathcal{H}_i$  with the private oracles  $\mathcal{H}'_i$  ( $i = 1, 2, 3$ ):

$$\begin{aligned} C_3 &= \mathcal{H}'_1(ID_i \| ID_S \| C_1 \| C_2); \\ C_4 &= \mathcal{H}'_2(ID_i \| ID_S \| C_1 \| C_2); \\ sk_U &= sk_S = \mathcal{H}'_3(ID_i \| ID_S \| C_1 \| C_2) \end{aligned}$$

As a result, the values of  $C_3, C_4, sk_U, sk_S$  are completely independent from  $k, K_U$  and  $K_S$ .  $\mathbf{G}_7$  and  $\mathbf{G}_6$  are indistinguishable unless the event  $\text{AskH}_7$  occurs:

$$|\Pr[\text{Succ}_7] - \Pr[\text{Succ}_6]| \leq \Pr[\text{AskH}_7] \quad (9)$$

$$|\Pr[\text{AskPara}_7] - \Pr[\text{AskPara}_6]| \leq \Pr[\text{AskH}_7] \quad (10)$$

$$|\Pr[\text{AskAuth}_7] - \Pr[\text{AskAuth}_6]| \leq \Pr[\text{AskH}_7] \quad (11)$$

*Lemma 1:* The probabilities of the events **Succ<sub>7</sub>** and **AskPara<sub>7</sub>** in this game can be up-bounded by:

$$|\Pr[\text{Succ}_7]| = \frac{1}{2} \quad |\Pr[\text{AskPara}_7]| \leq C' \cdot q_{send}^{s'} + \frac{q_{send}}{2^{l_0}} \quad (12)$$

*Proof.* In the game **G<sub>7</sub>**, the session keys are computed with private hash oracle unknown to  $\mathcal{A}$ , and thus  $\Pr[\text{Succ}_7] = \frac{1}{2}$ .

Let us denote by  $R(U)$  the set of  $(C_2, C_3)$  received by a client instance, and by  $R(S)$  the set of  $C_4$  used by a server instance. Since we have avoided the cases where  $\mathcal{A}$  have been lucky in guessing  $k$ ,  $\mathcal{A}$  can correctly compute  $k$  with the help of either a  $\text{Corrupt}(I = U^i, 1)$ -query or a  $\text{Corrupt}(I = U^i, 2)$ -query, the probability of which is denoted by  $\Pr[\text{AskPara}_7\text{WithCorr}_1]$  and  $\Pr[\text{AskPara}_7\text{WithCorr}_2]$ , respectively. As discussed in Sec. III of the main text, it is more desirable (realistic) to assume passwords to be Zipf-distributed [86] than to make the traditional, commonly used (yet unrealistic) assumption that passwords are uniformly distributed (see some notable literature [7], [87]–[90]). From an information theoretical point of view, since we have avoided collisions in the Game **G<sub>2</sub>**,

$$\begin{aligned} & |\Pr[\text{AskPara}_7\text{WithCorr}_1]| \\ &= \Pr[\exists C_3 \in R(U), (1, ID_i \| ID_S \| Y_1 \| C_2 \| k \| *, C_3) \in \Lambda_{\mathcal{A}}] \\ &+ \Pr[\exists C_4 \in R(S), (0, *, P_i) \in \Lambda_{\mathcal{A}}] \leq C' \cdot q_{send}^{s'} \quad (13) \end{aligned}$$

$$\Pr[\text{AskPara}_7\text{WithCorr}_2] \leq \frac{q_{send}}{2^{l_0}} \quad (14)$$

**Game G<sub>8</sub>:** In this game, we simulate the executions using the random self-reducibility of the Diffie-Hellman problem [91], given one CDH instance  $(A, B)$ . Note that, we do not need to know the values of  $\theta$  and  $\varphi$ , since the values of  $K_U$  and  $K_S$  are no longer needed to compute the authenticators or session keys:

- ▶ **Rule U1<sup>(8)</sup>** – chooses a random number  $\alpha \in Z_p^*$  and computes  $C_1 = A^\alpha \bmod p$ ,  $Y_1 = y^\alpha \bmod p$ ,  $k = \mathcal{H}_0(x \| ID_i \| T_{reg}) = N_i \oplus \mathcal{H}_0(b \| PW_i)$ ,  $CID_i = ID_i \oplus \mathcal{H}_0(C_1 \| Y_1)$ ,  $CAK_i = (a_i \| k) \oplus \mathcal{H}_0(Y_1 \| C_1)$  and  $M_i = \mathcal{H}_0(Y_1 \| k \| CID_i)$ . Also add the record  $(C_1, \alpha)$  in  $\Lambda_{\mathcal{A}}$ .
- ▶ **Rule S2<sup>(8)</sup>** – chooses a random number  $\beta \in Z_p^*$  and computes  $C_2 = B^\beta$  and  $C_3 = \mathcal{H}'_1(ID_i \| ID_S \| C_1 \| C_2)$ . Also adds the record  $(C_2, \beta)$  in  $\Lambda_B$ .

$$\Pr[\text{AskH}_7] = \Pr[\text{AskH}_8] \quad (15)$$

Remember that **AskH<sub>8</sub>** means that the adversary  $\mathcal{A}$  had queried the random oracles  $\mathcal{H}_i (i = 1, 2, 3)$  on  $(ID_i \| ID_S \| Y_1 \| C_2 \| * \| CDH(C_1, C_2))$ . By picking randomly in the  $\Lambda_{\mathcal{A}}$ -list we can get the Diffie-Hellman secret value with probability  $\frac{1}{q_h}$ . This is a triple  $(C_1, C_2, \text{CDH}(C_1, C_2))$ . We can then simply look in the lists  $\Lambda_A$  and  $\Lambda_B$  to find the values  $\alpha$  and  $\beta$  such that  $C_1 = A^\alpha$  and  $C_2 = B^\beta$ :

$$\text{CDH}(C_1, C_2) = \text{CDH}(A^\alpha, B^\beta) = \text{CDH}(A, B)^{\alpha\beta}$$

and thus:

$$|\Pr[\text{AskH}_8]| \leq q_h \text{Adv}_{\mathcal{P}}^{\text{CDH}}(t') \quad (16)$$

where  $t' \leq t + (q_{send} + q_{exe} + 1) \cdot \tau_e$ .

**Conclusion of the proof:** By combining above equations, one gets the announced result. Firstly, from Eqs.(1)-(5) we get:

$$|\Pr[\text{Succ}_4] - \Pr[\text{Succ}_0]| \leq \frac{(q_{send} + q_{exe})^2}{2p} + \frac{q_h^2}{2^{l+1}} + \frac{q_{send}}{2^l}.$$

Secondly, from Eqs.(6)-(9) we get:

$$|\Pr[\text{Succ}_7] - \Pr[\text{Succ}_4]| \leq \Pr[\text{AskPara}_5] + \Pr[\text{AskAuth}_6] + \Pr[\text{AskH}_7].$$

Thirdly, from the definition we know:

$$\Pr[\text{AskAuth}_6] \leq \Pr[\text{AskH}_6].$$

Finally, based on Eqs.(10)-(16) we get:

$$\begin{aligned} \text{Adv}_{\mathcal{P}}^{\text{ake}}(\mathcal{A}) &= 2\Pr[\text{Succ}_7] - 1 + 2(\Pr[\text{Succ}_0] - \Pr[\text{Succ}_7]) \\ &\leq C' \cdot q_{send}^{s'} + 12q_h \text{Adv}_{\mathcal{P}}^{\text{CDH}}(t') \\ &\quad + \frac{q_h^2 + 6q_{send}}{2^l} + \frac{(q_{send} + q_{exe})^2}{p}, \end{aligned}$$

where we use the Zipf model of Tianya in [86], where  $C' = 0.062239$  and  $s' = 0.155478$ ;  $n_0 = 2^8$ ;  $t' \leq t + (q_{send} + q_{exe} + 1) \cdot \tau_e$  and  $l = \min\{l_i\}, i = 0, 1, 2, 3$ .  $\square$

## B. Proof of Theorem 2

*Proof.* The proof is similar to that of Theorem 1.

Firstly, we define an additional event:

- **Auth<sub>n</sub>** occurs if  $\mathcal{A}$  correctly guesses the authenticator  $C_3$  or  $C_4$  that will be accepted by the corresponding party and that has been built by the adversary herself in game **G<sub>n</sub>**,  $n = 0, 1, \dots, 7$ .

Thus, we define

$$\text{Adv}_{\mathcal{P}}^{\text{auth}}(\mathcal{A}) = \Pr[\text{Auth}_0]$$

Secondly, we use the same sequence of games presented in the previous section, and extend Eqs. (1)-(7) to obtain:

$$\begin{aligned} |\Pr[\text{Auth}_1] - \Pr[\text{Auth}_0]| &= 0 \\ |\Pr[\text{Auth}_2] - \Pr[\text{Auth}_1]| &\leq \frac{(q_{send} + q_{exe})^2}{2p} + \frac{q_h^2}{2^{l+1}} \\ |\Pr[\text{Auth}_3] - \Pr[\text{Auth}_2]| &\leq \frac{q_{send}}{2^l} \\ |\Pr[\text{Auth}_4] - \Pr[\text{Auth}_3]| &\leq \frac{q_{send}}{2^l} \\ |\Pr[\text{Auth}_5] - \Pr[\text{Auth}_4]| &\leq \Pr[\text{AskPara}_5] \\ |\Pr[\text{Auth}_6] - \Pr[\text{Auth}_5]| &\leq \Pr[\text{AskAuth}_6] \leq 2\Pr[\text{AskH}_7] \\ \Pr[\text{Auth}_7] &= \Pr[\text{Auth}_6] = 0 \end{aligned}$$

Thus, we have

$$\begin{aligned} \text{Adv}_{\mathcal{P}}^{\text{auth}}(\mathcal{A}) &\leq C' \cdot q_{send}^{s'} + 5q_h \text{Adv}_{\mathcal{P}}^{\text{CDH}}(t + (q_{send} + q_{exe} \\ &\quad + 1) \cdot \tau_e) + \frac{q_h^2 + 6q_{send}}{2^{l+1}} + \frac{(q_{send} + q_{exe})^2}{2p}. \quad \square \end{aligned}$$

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